A Three-Node Packet Radio Network

MOSHE SIDI AND ADRIAN SEGALL

Abstract—**A two-hop packet radio network consisting of three nodes is considered. The three nodes have infinite buffers and share a common radio channel for transmitting their packets. Two of the nodes forward their packets to a third node that acts as a source of data as well as a relay that forwards all the packets entering the network to a main station. We assume that two of the nodes are granted full rights in accessing the channel while the third node uses a random access scheme. For this network we derive the condition for steady state and the generating function of the joint queue length distribution at the nodes in steady state. We also give several numerical examples and compare the performance of the network with and without a relay node.**

I. Introduction

The purpose of this paper is to analyze the three-node packet-radio network with a configuration depicted in Fig. 1. This two-hop network is an extension of the one-hop network considered in [1] (Fig. 2), where two radio nodes transmit their data directly to a common receiver. The three nodes of our system are assumed to have *infinite buffers* and use a common radio channel to transmit their packets. Time is divided into slots corresponding to the transmission time of a packet and a node may start packet transmissions only at the beginning of a slot.

In the network of Fig. 1, nodes 2 and 3 send their packets to node 1, which forwards all packets to a main station. One important aspect of this paper is that it enables us to analytically evaluate the performance degradation due to the addition of a relay to the network.

We assume here that the transmissions of node 1 do not interfere with the transmissions of the other nodes, i.e., the station is out of the transmission range of nodes 2 and 3. When nodes 2 or 3 transmit, only node 1 can hear them. When they both transmit a packet at the same time, a collision occurs and the transmitted packets are not received at node 1. In addition we assume that node 1 is not able to transmit and receive a packet (transmitted by other nodes) at the same time. All nodes share a common channel and their channel access schemes will be specified in Section II. In Section III we briefly summarize the steady-state analysis (for details the interested reader is referred to [2]). The performance of the systems with and without a relay node (the present paper versus [1]) is compared in Section IV for independent Bernoulli arrival processes.

II. The Model

Packets arrive at the three nodes (Fig. 1) that have infinite buffers from the outside of the system, and in general the arrival processes may be correlated. Let $A_i(t)$, $1 \leqslant i \leqslant 3$ be the number of packets entering node i from its corresponding source (outside of the system) in the time interval $(t, t+1]$. The input process $\{A_i(t)\}_{i=1}^{3}$ is assumed to be a sequence of independent and identically distributed random vectors with integer-valued elements. Let

$$F(z_3, z_2, z_1) = E\{z_3{}^{A_3(t)} z_2{}^{A_2(t)} z_1{}^{A_1(t)}\}. \qquad (1)$$

During each slot that their queues are not empty, nodes 1 and 2 transmit the packet at the head of their queues. The transmissions of node 3 are randomized, so that during each slot that its queue is not empty, it transmits the packet at the head of its queue with probability p. The transmissions of nodes 2 and 3 might be unsuccessful either because they transmit simultaneously or because node 1 is transmitting at the same time and, therefore, is not able to receive packets transmitted by nodes 2 or 3. We assume that nodes 2 and 3 are able to detect at the end of the slot if their transmissions were successful. In case of unsuccessful transmission at any node, the unsuccessfully transmitted packet remains at the head of the queue and must be retransmitted by the node according to the schemes presented above.

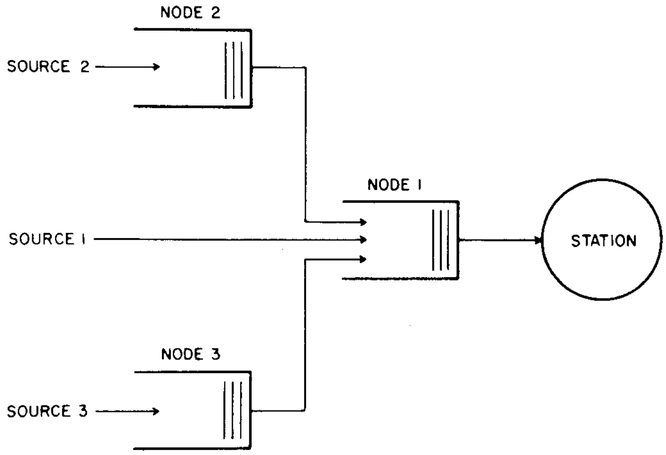

Fig. 1. A three-node packet-radio network.

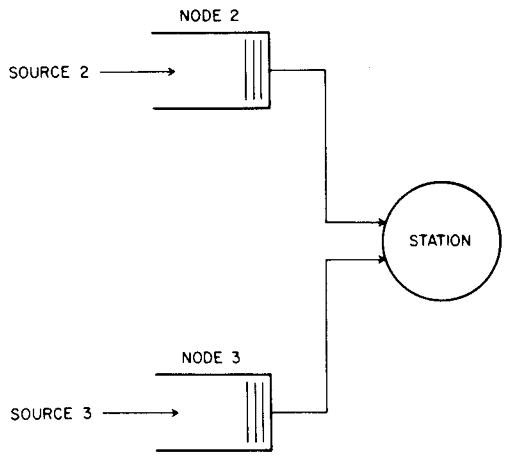

Fig. 2. The two-node network considered in [1].

Paper approved by the Editor for Computer Communication of the IEEE Communications Society for publication after presentation at INFOCOM '83, San Diego, CA, April 1983. Manuscript received December 7, 1982; revised March 16, 1984.

M. Sidi is with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel.

A. Segall is with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel, on leave at the IBM Thomas J. Watson Research Center, Yorktown Heights, NY 10598.

0090-6778/84/1200-1336$01.00 © 1984 IEEE

III. STEADY-STATE ANALYSIS

Let $L_i(t)$, $1 \leqslant i \leqslant 3$, be the number of packets at node i at time t and let

$$G(z_3, z_2, z_1) = \lim_{t \to \infty} E\{z_3{}^{L_3(t)} z_2{}^{L_2(t)} z_1{}^{L_1(t)}\} \tag{2}$$

be the steady-state generating function of the joint queue length probability distribution at the nodes. Then, using a standard technique, it can be shown that (here $\bar{p} = 1 - p$)

$$\begin{aligned} G(z_3, z_2, z_1) &= F(z_3, z_2, z_1)\{G(0, 0, 0) + [G(0, z_2, 0) \\ &\quad - G(0, 0, 0)] z_2{}^{-1} z_1 + [G(z_3, 0, 0) - G(0, 0, 0)] \\ &\quad \cdot (p z_3{}^{-1} z_1 + \bar{p}) + [G(z_3, z_2, 0) - G(0, z_2, 0) \\ &\quad - G(z_3, 0, 0) + G(0, 0, 0)] (\bar{p} z_2{}^{-1} z_1 + p) \\ &\quad + [G(z_3, z_2, z_1) - G(z_3, z_2, 0)] z_1{}^{-1}\}. \end{aligned} \tag{3}$$

In (3) we encounter a phenomenon common to interfering queues [1], as well as to other queueing systems with dependent queues [3], [4], that the joint generating function is expressed in terms of several boundary terms. Here to determine $G(z_3, z_2, z_1)$ we still have to determine the boundary functions $G(0, z_2, 0)$, $G(z_3, 0, 0)$, $G(z_3, z_2, 0)$ and the boundary constant $G(0, 0, 0)$. The detailed derivation of these boundary terms may be found in [2]. Here we give a brief summary of the results.

Let σ be the unique solution of the equation $\sigma = F(0, \sigma^2, \sigma)$ in the unit circle $|\sigma| < 1$ and for $|z_2| < 1$ let $f_0(z_2)$ be the unique solution of the equation $f_0(z_2) = f(0, z_2, f_0(z_2))$ in the unit circle $|f_0(z_2)| < 1$. The existence and uniqueness of these solutions follow from Rouche's theorem [6]. Then,

$$G(0, 0, 0) = \frac{p(\bar{p} - r_2) - \bar{p} r_3 - p\bar{p}(r_1 + r_2 + r_3)}{p\left\{1 - p\dfrac{1 - f_0(1) + f_0(1)(\sigma^{-1} - 1)/\sigma}{f_0{}^{-1}(1) - f_0(1)}\right\}} \tag{4}$$

where

$$r_i = \left.\frac{\partial F(z_3, z_2, z_1)}{\partial z_i}\right|_{z_3 = z_2 = z_1 = 1} \qquad 1 \leqslant i \leqslant 3 \tag{5}$$

and

$$G(0, z_2, 0) = \frac{G(0, 0, 0)(1 - z_2{}^{-1} f_0(z_2)) + p f_0(z_2) G_3{}'(0, 0, 0)}{f_0{}^{-1}(z_2) - z_2{}^{-1} f_0(z_2)} \tag{6}$$

where

$$G_3{}'(0, 0, 0) = \frac{\sigma^{-1} - 1}{p\sigma} G(0, 0, 0). \tag{7}$$

Let $\hat{z}_2(z_1) \triangleq \bar{p} z_1{}^2/(1 - z_1 p)$. For $|z_3| < 1$ let $f_1(z_3)$ be the unique solution of $f_1(z_3) = F(z_3, \hat{z}_2(f_1(z_3)), f_1(z_3))$ in the unit circle $|f_1(z_3)| < 1$. Then,

$$G(z_3, 0, 0) = p\,\frac{G(0, 0, 0)[1 - z_3{}^{-1} f_1] + [G(0, \hat{z}_2(f_1), 0) - G(0, 0, 0)]\,[\hat{z}_2{}^{-1}(f_1) f_1 - 1]}{[\bar{p}\hat{z}_2{}^{-1}(f_1) - p z_3{}^{-1}] f_1 - 1 + 2p}. \tag{8}$$

Finally, for $|z_2| < 1, |z_3| < 1$, let $f_2(z_3, z_2)$ be the unique solution of the equation $f_2(z_3, z_2) = F(z_3, z_2, f_2(z_3, z_2))$ in the unit circle $|f_2(z_3, z_2)| < 1$. Then,

$$G(z_3, z_2, 0) = \frac{H_1(z_3, z_2)}{H_2(z_3, z_2)} \tag{9}$$

where

$$\begin{aligned} H_1(z_3, z_2) &= G(0, 0, 0)[1 - z_3{}^{-1} f_2] p + [G(0, z_2, 0) \\ &\quad - G(0, 0\ 0)](z_2{}^{-1} f_2 - 1) p \\ &\quad + G(z_3, 0, 0)[(p z_3{}^{-1} - \bar{p} z_2{}^{-1}) f_2 + 1 - 2p] \end{aligned} \tag{10a}$$

and

$$H_2(z_3, z_2) = f_2{}^{-1} - p - \bar{p} z_2{}^{-1} f_2. \tag{10b}$$

Notice that the condition for steady state is $G(0, 0, 0) > 0$.

The average number of packets at node i ($1 \leqslant i \leqslant 3$) in steady state is given by

$$\bar{L}_i = \left.\frac{\partial G(z_3, z_2, z_1)}{\partial z_i}\right|_{z_3 = z_2 = z_1 = 1} \qquad 1 \leqslant i \leqslant 3. \tag{11}$$

In addition, by applying Little's law [5] to each node in the network, we can also derive the average time delays at the nodes. These quantities are given by

$$T_1 = \frac{\bar{L}_1}{r_1 + r_2 + r_3}; \quad T_i = \frac{\bar{L}_i}{r_i} \qquad i = 2, 3. \tag{12}$$

Finally, the total average delay in the network is given by

$$T = \frac{\bar{L}_1 + \bar{L}_2 + \bar{L}_3}{r_1 + r_2 + r_3}. \tag{13}$$

IV. INDEPENDENT BERNOULLI ARRIVAL PROCESSES

Although the analysis in the previous section has been done for general arrival processes, some of the expressions become simpler for independent Bernoulli arrival processes, i.e., for

$$F(z_3, z_2, z_1) = (z_3 r_3 + \bar{r}_3)(z_2 r_2 + \bar{r}_2)(z_1 r_1 + \bar{r}_1). \tag{14}$$

The expressions for $\bar{L}_i$, $i = 1, 2, 3$, are too complicated to be given here and the interested reader can find them in [2]. The condition for steady state in the network is

$$p(1 - r_1 - r_2 - r_3 - r_2/\bar{p}) > r_3. \tag{15}$$

Equation (15) can be explained intuitively as follows. Assume heavy traffic conditions, i.e., node 3 almost always has packets for transmission. It is clear that $r_1 + r_2 + r_3$ is the fraction of time node 1 holds the channel and $r_2/\bar{p}$ is the fraction of time node 2 holds the channel. Only when both nodes 1 and 2 do not hold the channel, node 3 may succeed in its transmission. Since it tries to transmit with probability p, packets may leave it with a rate of at most $p(1 - r_1 - r_2 - r_3 - r_2/\bar{p})$. Since the arrival rate to any node must be less than the departure rate from that node, we obtain (10).

As expected, we have found that the average time delay at node 1 is unaffected by the value of p—the transmission probability at node 3, since it cannot interfere with the transmissions of node 1. In Figs. 3–5 we plot T_2, T_3, and T, respectively

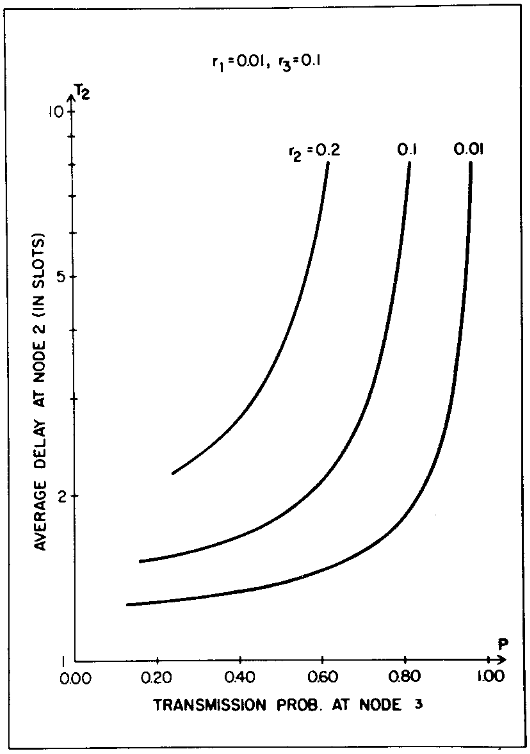

Fig. 3. Average delay at node 2 versus the transmission probability at node 3.

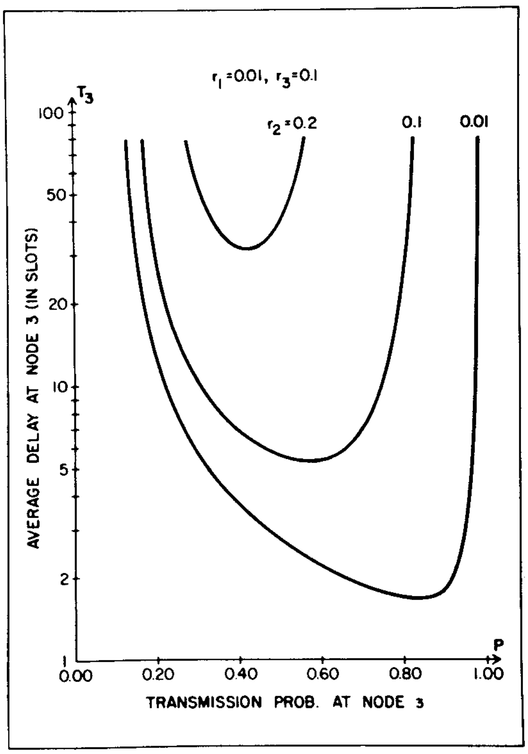

Fig. 4. Average delay at node 3 versus the transmission probability at node 3.

versus p—the transmission probability at node 3, for $r_1 = 0.01$, $r_3 = 0.1$, and r_2 ranging from 0.01 to 0.2. As expected, the average delay at node 2 increases when p increases, since then its transmissions collide more frequently with transmissions from node 3. More interesting is the behavior of the average delay at node 3. Here p has some value for which T_3 is minimized (for given r_1, r_2, and r_3). When p either increases or decreases from this value, T_3 increases. The reason is that when p

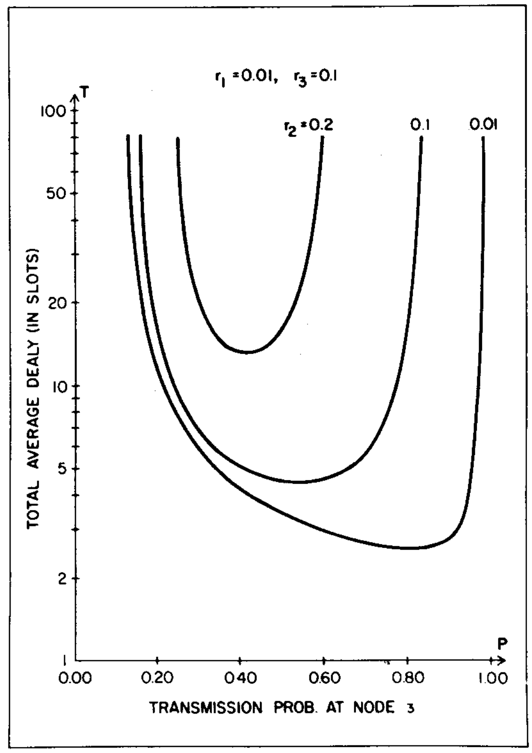

Fig. 5. Total average delay versus the transmission probability at node 3.

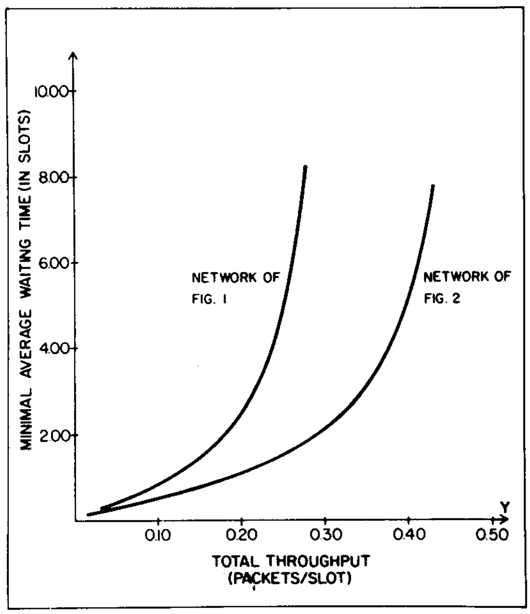

Fig. 6. Minimal total average delay versus the total throughput.

becomes small, node 3 attempts to transmit relatively rarely, so its queue increases. When p becomes large, then node 3 attempts to transmit more frequently, thus interfering with the transmissions of node 2, and the queue lengths at both nodes are large. As we see from Fig. 5, the parameter p is a very critical design parameter of this system, and for given values for r_1, r_2, and r_3, there exists an optimal p that minimizes the total average delay in the network.

In order to understand the effect of adding the intermediate node (node 1) in the network of Fig. 1 compared with the network of Fig. 2, we shall assume that $r_1 = 0$, i.e., no packets arrive at node 1 from its corresponding source and it serves only as a relay node. In this case, from (15) we see that when $r_2 = r_3 = r$, the total throughput of the network γ ($\gamma = 2r$) should be less than 1/3 in the network with the relay node

(Fig. 1) while $\gamma < 1/2$ [1] without such a node (Fig. 2). Consequently, the addition of a relay node decreases the maximal throughput by 33 percent in this case.

In Fig. 6, the minimal average *waiting time* of packets in the network is plotted versus the throughput for equal arrival rates at nodes 2 and 3 for the network with and without the relay node. This minimal average waiting time is obtained from the minimal average delay time minus one unit for the network of Fig. 2, and minus two units for the network of Fig. 1. As is seen from Fig. 6, the addition of a relay node significantly deteriorates the performance of the network.

References

[1] M. Sidi and A. Segall, "Two interfering queues in packet-radio networks," *IEEE Trans. Commun.*, vol. COM-31, pp. 123–127, Jan. 1983.

[2] ——, "A three-node packet-radio network," in *Proc. INFOCOM'83,* San Diego, CA, Apr. 1983, pp. 222–228.

[3] G. Fayolle and R. Iasnogorodski, "Two-coupled processors—The reduction to a Riemann-Hilbert problem," *Wahrscheinhilchkeitstheorie,* pp. 1–27, 1979.

[4] M. Eisenberg, "Two queues with alternating service," *SIAM J. Appl. Math.,* vol. 36, pp. 287–303, Apr. 1979.

[5] J. D. C. Little, "A proof for the queueing formula $L = \lambda W$," *Oper. Res.*, vol. 9, pp. 383–387, 1961.

[6] E. T. Copson, *Theory of Functions of a Complex Variable.* London, England: Oxford Univ. Press, 1948.