Discrete-Time Priority Queues with Partial Interference

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Abstract—A class of discrete-time priority queueing systems with partial interference is considered. In these systems, N nodes share a common channel to transmit their packets. One node uses a random access scheme, while other nodes access the channel according to preassigned priorities. Packet arrivals are modeled as discrete-time batch processes, and packets are forwarded through the network according to fixed prescribed probabilities.

Steady-state analysis of the class of systems under consideration is provided. In particular, we present a recursive method for the derivation of the joint generating function of the queue lengths distribution at the nodes in steady state. The condition for steady state is also derived. A simple example demonstrates the general analysis and provides some insights into the behavior of systems with partial interference such as multihop packet radio systems.

I. INTRODUCTION

THE survey paper by Kobayashi and Konheim [1] discusses many models of discrete-time queueing systems. Such systems have been receiving increased attention in recent years [2]-[4] due to their usefulness in modeling and analyzing various types of communication systems. Packet-switched communication networks with point-to-point links between the nodes, where data packets are of a fixed length, motivated most of these models. The models in [2]-[4] are of a tandem nature since in point-to-point networks the transport of a packet from its source to its destination involves the transmission of the packet over a succession of links. The fixed packet length assumption induces the discrete-time nature of the models.

In this paper, we consider a class of discrete-time priority queueing systems with partial interference. Consideration of these systems has been motivated primarily by the class of packet-switched communication networks called the multiaccess/broadcast networks, or packet radio networks. In these communication networks, all nodes share a common channel through which they transmit their packets and from which they extract packets destined to them, hence the multiaccess nature of these networks. In

addition, when a node transmits a packet through the shared channel, all nodes that are within its transmission range hear this transmission, thus inducing the broadcast nature of the system.

We assume that the channel time axis is slotted into intervals of size equal to the transmission time of a packet. All packets are assumed to be of fixed and equal size. The nodes are synchronized so that they may start transmission of a packet only at the beginning of a slot, hence the discrete-time nature of the system. All nodes are assumed to have *infinite* buffers.

One of the most crucial issues in multiaccess networks is the protocol required to transmit packets on a shared channel in a distributed environment. For a survey of multiaccess protocols, the reader is referred to [5]. The design and analysis of multiaccess protocols is not trivial. This is due to the following two facts that hold for packet radio networks: 1) If two or more nodes transmit packets during the same slot to the same node, then the overlap in transmission destroys all packets involved in the transmission, and 2) a transmitting node is unable to receive packets transmitted by other nodes of the system. These two facts, together with the broadcast nature of the network, give rise to *statistical dependence* between the queues at the nodes of the network. In most cases, this dependence is rather complicated, and therefore, there is little hope of obtaining analytical results for general multiaccess protocols and for general network configurations. The purpose of this paper is to analyze a rather general network configuration with a specific mode of operation.

One mode of operation that can be accomplished in multiaccess networks is a conflict-free mode. This mode of operation is known to be very attractive, as it commonly provides high channel efficiency. Conflict-free operation can be achieved if every node knows perfectly which are the nodes that have packets ready for transmission at the beginning of each slot. This is possible in systems that have a central scheduler that schedules the transmissions according to information it receives from the nodes or in systems where the nodes exchange this information among themselves. Some examples of conflict-free protocols have been described in the literature [6]-[8]. Generally, any order of transmission can be used, in particular, fixed priority [6], [7] as well as alternating priority [7], [8] can be easily implemented. The essential assumption in devising conflict-free protocols is that all nodes of the system can hear each other. Yet, if there are

some nodes that cannot exchange information with the scheduler or with other nodes, on which nodes have packets ready for transmission, then their transmissions cannot be accommodated in a conflict-free mode of operation. Such situations are encountered when some nodes are hidden from other nodes in the system [9]-[10]. Commonly, nodes are hidden from others in a multihop environment where packets must traverse more than one hop in order to get to their destinations (see the example in Section IV). In such situations, the hidden nodes that are not able to participate in the conflict-free protocol should use some random access scheme [5].

The class of discrete-time queueing systems that we consider in this paper consists of systems having $N - 1$ nodes that access the channel in a conflict-free mode according to fixed priorities that are preassigned to them. No two nodes have the same priority, and a given node is allowed to use the channel in a given slot only if it has a packet ready for transmission and all nodes with higher priority have empty queues. In addition, there is an extra node in the system that cannot be accommodated in the conflict-free mode of operation and therefore is allowed to use the channel in any slot on a random basis. If the node uses the channel along with any other node, then their packets are destroyed and must be retransmitted, hence the interfering feature of the systems under consideration. The assumption of a single interfering node is quite restrictive. However, as we shall indicate later, the analysis of systems having multiple interfering nodes is formidable, if possible at all. The single interfering node assumption enables us to analyze the system exactly and derive some insight into the behavior of interfering systems. Furthermore, it would probably be possible to approximate systems with multiple interfering nodes by aggregating them into a single node and then use the results of this paper. The latter idea is still to be explored.

To enhance the network structure of the problem, we attach to each node a given probability distribution that indicates the probability that a packet transmitted by the node is forwarded to one of the other nodes or to the outside of the system.

Outside sources feed the nodes of the system with new packets. An important feature of this paper is that these sources are allowed to depend on each other. Thus, we are able to characterize a rather general class of batch arrival processes.

Several discrete-time queueing systems that have been previously investigated [11]-[13] are related to the class of systems considered in this paper. In [11], a "loop system," in which nodes transmit packets only to the outside of the system, the arrival processes are independent, and there is no interference, has been considered. In [4] and [12], two-node systems have been analyzed, and in [13] no interference is allowed.

The paper is organized as follows. In Section II, we describe the model along with the assumptions and several definitions and notations used throughout the paper. In Section III, we present the steady-state analysis of the class of systems under consideration. In particular, we develop a method for deriving the joint generating function of the queue lengths at the nodes, and we give the ergodicity condition for the system. Moments of the queue lengths at the nodes can be derived from the generating function, and average time delays can be obtained by using Little's law [14]. In Section IV, we give an example that demonstrates the general analysis and provides some insight into the behavior of systems with partial interference. Finally, in Section V, we summarize and discuss some of the ramifications and limitations of our model and our results.

II. MODEL DESCRIPTION

We consider a discrete-time queueing system in which the time axis is divided into intervals of equal size, referred to as slots. The slots correspond to the transmission time of a packet, and all packets are assumed to be of the same fixed size. The system consists of N nodes, and packets arrive randomly to the nodes from N sources that in general may be correlated. Let $A_i(t)$, $i = 1, 2, \cdots$, N, $t = 0, 1, 2, \cdots$, be the number of packets entering node i from its corresponding source during the time interval $(t, t + 1)$. The input process $\{A_i(t)\}_{i=1}^{N}$, $t = 0, 1, 2, \cdots$, is assumed to be a sequence of independent and identically distributed random vectors with integer-valued elements. Let the corresponding probability distribution and generating function of the input processes be

$$a(i_1, i_2, \cdots, i_N) = \Pr \left\{ A_1(t) = i_1, A_2(t) \right.$$

$$= i_2, \cdots, A_N(t) = i_N \right\}$$

$$i_j = 0, 1, 2, \cdots, \quad 1 \le j \le N$$

(1a)

$$F(z) = E \left\{ \prod_{i=1}^{N} z_i^{A_i(t)} \right\}$$

(1b)

where we use the notation $z = (z_1, z_2, \cdots, z_N)$.

All nodes share a common channel for transmission of their packets, and transmissions are started only at the beginning of a slot. No more than one packet may be transmitted in any given time slot by a single node. Using some conflict-free protocol, the channel is made available to nodes $i = 1, 2, \cdots, N - 1$ according to a fixed priority. Specifically, node $i (1 \le i \le N - 1)$ transmits the packet at the head of its queue whenever the queues at nodes 1, 2, $\cdots$, $i - 1$ are empty and the one at node i is nonempty. Node N is a special node that cannot participate in the conflict-free protocol and therefore apply a random access protocol. At the beginning of each slot for which the queue at node N is nonempty, a coin with probability of success p is tossed. In the case of a success, node N transmits the packet at the head of its queue; otherwise, it remains silent. Whenever node N transmits while another node $i (1 \le i \le N - 1)$ is also transmitting, then both transmissions are unsuccessful and the two nodes

must retransmit the packets at the head of their queues according to the protocols described above.

In any case, when a node $i(1 \leq i \leq N)$ transmits a packet successfully, then the packet joins node $j(1 \leq j \leq N, j \neq i)$ with probability $\theta_i(j)$ or leaves the system with probability $\theta_i(0)$. We assume here that $\theta_i(i) = 0$. All packets received by a node from an outside source or from other nodes are buffered in a common outgoing queue and transmitted on a first-come–first-served basis. It is assumed that packets indeed arrive at every node of the system, so that there is no node that is empty with probability 1 (in other words, nodes that are always empty are ignored). Finally, we assume that the buffers at the nodes are *infinite*. A schematic figure of a node i in the system is depicted in Fig. 1.

III. STEADY-STATE ANALYSIS

To describe the evolution of the queue contents at the nodes, we need several definitions. Let $L_i(t)$, $1 \leq i \leq N$, $t = 0, 1, 2, \cdots$, be the number of packets at node i at time t and let $U(L_i(t))$ $(1 \leq i \leq N, t = 0, 1, 2, \cdots)$ be a binary-valued random variable that takes value 1 if $L_i(t) > 0$ and 0 otherwise. Let V be a binary-valued random variable that takes values 1 and 0 with probabilities p and $\bar{p} = 1 - p$, respectively. Also, let $D_i^j(t)$, $1 \leq i \leq N, 0 \leq j \leq N, t = 0, 1, 2, \cdots$, be a binary-valued random variable that takes value 1 if a packet is successfully transmitted from node i to node j at time t where $j = 0$ corresponds to the case that the packet leaves the system.

Using these definitions, it is easy to see that the system under consideration evolves for $t = 0, 1, 2, \cdots$ as follows. For $1 \leq i \leq N$,

$$L_i(t + 1) = L_i(t) + A_i(t) + \sum_{m=1}^{N} D_m^i(t)$$
$$- V_i(t) U(L_i(t)) \prod_{m=1}^{i-1} [1 - U(L_m(t))]$$

(2a)

where

$$V_i(t) = \begin{cases} 1 - VU(L_N(t)) & 1 \leq i \leq N - 1 \\ V & i = N. \end{cases}$$

(2b)

Notice that $V_i(t)$ is a binary-valued random variable, and for $1 \leq i \leq N - 1$, it can be interpreted as the interference indicator at time t, i.e., it indicates whether or not node N interferes with the transmission of node i at time t. Clearly, $\{L_i(t)\}_{i=1}^{N}$ is a vector Markov chain. Assuming that this Markov chain is ergodic (we shall derive the condition for this later), let us consider the steady-state joint generating function of the queue lengths distribution,

$$G(z) = \lim_{t \to \infty} E\left\{ \prod_{i=1}^{N} z_i^{L_i(t)} \right\}.$$
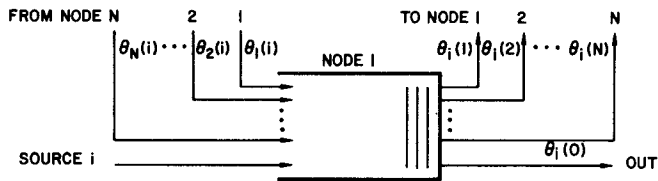
(3)

Fig. 1. An example of a node i in the system.

For notational convenience, let us define the following boundary generating functions:

$$G_0(z) = G(z) \tag{4a}$$

$$G_i(z) = G(z)\big|_{z_1 = z_2 = \cdots = z_i = 0} \qquad 1 \leq i \leq N \tag{4b}$$

$$\hat{G}_i(z) = G_i(z)\big|_{z_N = 0} \qquad 0 \leq i \leq N - 1. \tag{4c}$$

Notice that by our definition $G_N(z) = \hat{G}_{N-1}(z)$ is a constant representing the steady-state probability that the system will be empty. Finally, let us define the following polynoms:

$$Q_i(z) = \theta_i(0) + \sum_{m=1}^{N} \theta_i(m) z_m \qquad 1 \leq i \leq N. \tag{5}$$

Theorem 1: With the above notations, the following holds:

$$G(z) = F(z)\Big\{ G_N(z)$$
$$+ [G_{N-1}(z) - G_N(z)][\bar{p} + pz_N^{-1}Q_N(z)]$$
$$+ \sum_{i=1}^{N-1} [\hat{G}_{i-1}(z) - \hat{G}_i(z)] z_i^{-1} Q_i(z)$$
$$+ \sum_{i=1}^{N-1} [G_{i-1}(z) - G_i(z) - \hat{G}_{i-1}(z)$$
$$+ \hat{G}_i(z)][p + \bar{p}z_i^{-1}Q_i(z)]\Big\}. \tag{6}$$

The formal proof of Theorem 1 appears in Appendix A. Let us give here an intuitive explanation for (6). The right-hand side of (6) is a multiplication of the generating function of the joint arrival process, which by our assumptions is independent of the state of the system, and an expression that indicates, for the various states that the system may be in, which node is transmitting and how packets are moved within the network. Specifically, $G_N(z)$ corresponds to the case that the queues at all nodes are empty. $G_{N-1}(z) - G_N(z)$ corresponds to the situation that all nodes except node N are empty; therefore, with probability p a packet leaves node N and joins another node or leaves the system according to the probabilities $\theta_N(j)$, $0 \leq j \leq N$. $\hat{G}_{i-1}(z) - \hat{G}_i(z)$ for $1 \leq i \leq N - 1$ corresponds to the situation where node N is empty, as are nodes 1, 2, $\cdots$, $i - 1$, and node i has a packet for transmission. Then, a packet leaves node i and joins another node or leaves the system according to the prob-

abilities $\theta_i(j)$, $0 \leq j \leq N$. Finally, the term $G_{i-1}(z) - G_i(z) - \hat{G}_{i-1}(z) + \hat{G}_i(z)$ for $1 \leq i \leq N - 1$ corresponds to the case where nodes $1, 2, \cdots, i - 1$ are empty and nodes i and N have both packets for transmission. In this case, with probability p the two nodes interfere and no packet is moved, while if node N remains silent (this happens with probability $\bar{p} = 1 - p$), then a packet leaves node i and joins another node or leaves the system as before.

Rearranging (6) we obtain

$$G(z) = F(z) \frac{\sum_{i=1}^{N} H_i(z) G_i(z) + \sum_{i=0}^{N-1} \hat{H}_i(z) \hat{G}_i(z)}{1 - F(z)[p + \bar{p}z_1^{-1}Q_1(z)]} \quad (7a)$$

where

$$H_i(z) = \begin{cases} \bar{p}[z_{i+1}^{-1}Q_{i+1}(z) - z_i^{-1}Q_i(z)] & 1 \leq i \leq N - 2 \\ 1 - 2p + pz_N^{-1}Q_N(z) - \bar{p}z_{N-1}^{-1}Q_{N-1}(z) & i = N - 1 \\ p[1 - z_N^{-1}Q_N(z)] & i = N \end{cases} \quad (7b)$$

and

$$\hat{H}_i(z) = \begin{cases} -p[1 - z_1^{-1}Q_1(z)] & i = 0 \\ p[z_{i+1}^{-1}Q_{i+1}(z) - z_i^{-1}Q_i(z)] & 1 \leq i \leq N - 2 \\ p[1 - z_{N-1}^{-1}Q_{N-1}(z)] & i = N - 1. \end{cases} \quad (7c)$$

In (7), we encounter a common phenomenon in interfering queues, namely, that the generating function $G(z)$ is expressed in terms of several boundary functions. In order to uniquely determine $G(z)$ in (7), we will have to determine $2N - 1$ boundary functions,[1] $G_i(z)$, $1 \leq i \leq N$, and $\hat{G}_i(z)$, $0 \leq i \leq N - 2$. In what follows, we develop the method for obtaining these boundary functions. The basic idea is to first express $\hat{G}_i(z)$, $i = 0, 1, \cdots, N - 2$ (in this order) in terms of $\hat{G}_j(z)$, $i + 1 \leq j \leq N - 1$. Then $G_i(z)$, $i = 1, 2, \cdots, N - 1$, is expressed in terms of $\hat{G}_j(z)$, $0 \leq j \leq N - 1$, and $G_j(z)$, $i + 1 \leq j \leq N$. Finally, the constant $G_N(z)$ is determined from the normalization condition, and using backward substitutions all the boundary functions are determined. Along the above process, we mainly use the analytic properties of the generating function $G(z)$ in the polydisc $|z_i| \leq 1$, $1 \leq i \leq N$.

In order to proceed, we shall need the following lemma.

Lemma 1: Let $F(z)$ be the generating function of the joint arrival process (1b), $Q_1(z)$ be the function defined in (5) and $0 \leq p \leq 1$. Then, for given $|z_i| < 1$, $2 \leq i \leq N$, the following equation in z_1,

$$F(z)[pz_1 + (1 - p) Q_1(z)] = z_1 \quad (8)$$

has a unique solution $z_1 = z_1(z_2, z_3, \cdots, z_N)$ in the unit circle $|z_1| < 1$.

[1] Notice that in a general system where each node can interfere with any other node, we might have up to $2^N - 1$ boundary functions to determine. An example for such a system is a network in which all nodes use a random access policy.

Proof: Let $|z_1| = 1$ and $|z_i| < 1$, $2 \leq i \leq N$. We distinguish between two cases. The first is the case where packets do arrive to some node l, $2 \leq l \leq N$, from its corresponding source. The second is the case where no packets arrive to nodes $2 \leq l \leq N$ from their corresponding sources. Our assumption that packets do arrive to all nodes implies that in the latter case packets do arrive at node 1 from its corresponding source, and it routes some of them to at least one of the nodes l, $2 \leq l \leq N$.

Case 1: There exists some node $l(2 \leq l \leq N)$ for which the probability that a packet will arrive to it from its corresponding source is strictly positive, i.e., there exists $a(i_1, i_2, \cdots, i_N) > 0$ for some i_1 and some $i_l > 0$ $(2 \leq l \leq N)$. Therefore,

$$\left| F(z)[pz_1 + (1 - p) Q_1(z)] \right| \leq \left| F(z) \right|$$

$$= \left| \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \cdots \sum_{i_N=0}^{\infty} a(i_1, i_2, \cdots, i_N) \prod_{j=1}^{N} z_j^{i_j} \right|$$

$$\leq \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \cdots \sum_{i_N=0}^{\infty} a(i_1, i_2, \cdots, i_N) |z_l^{i_l}|$$

$$< \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \cdots \sum_{i_N=0}^{\infty} a(i_1, i_2, \cdots, i_N) = 1 = |z_1|.$$

$$(9)$$

Hence, applying Rouche's theorem [15], the claim is proved in this case.

Case 2: Packets arrive at node 1, and it routes some of them to at least one of the nodes $l(2 \leq l \leq N)$, i.e., there exists $\theta_1(l) > 0$ for some $2 \leq l \leq N$. Therefore,

$$\left| F(z)[pz_1 + (1 - p) Q_1(z)] \right| \leq \left| p + (1 - p) Q_1(z) \right|$$

$$= \left| p + (1 - p) \left[\theta_1(0) + \sum_{i=2}^{N} \theta_1(i)z_i \right] \right| < p$$

$$+ (1 - p) = 1 = |z_1|. \quad (10)$$

Hence, applying Rouche's theorem, the proof is completed. □

Let $\sigma_1(z_2, z_3, \cdots, z_N)$ (for simplicity σ_1) denote the unique solution of (8). Let $z^{(1)}$ denote the vector z with its first component z_1 replaced by σ_1. Using a proof sim-

ilar to that for Lemma 1, we can show that for $|z_i| < 1$, $3 \leq i \leq N$, the following equation in z_2,

$$F(z^{(1)})[pz_2 + (1 - p) Q_2(z^{(1)})] = z_2 \qquad (11)$$

has a unique solution in the unit circle $|z_2| < 1$. Let $\sigma_2(z_3, z_4, \cdots, z_N)$ denote this solution and $z^{(2)}$ denote the vector z with its first component z_1 replaced by $\sigma_1(\sigma_2(z_3, z_4, \cdots, z_N), z_3, \cdots, z_N)$ and its second component z_2 replaced by $\sigma_2(z_3, z_4, \cdots, z_N)$. Continuing this procedure, we have the following lemma that recursively determines the unique functions $\sigma_i(z_{i+1}, z_{i+2}, \cdots, z_N)$ $2 \leq i \leq N - 1$ as follows.

Lemma 2: With the above notations and for $2 \leq i \leq N - 1$, the following equation in z_i,

$$F(z^{(i-1)})[pz_i + (1 - p) Q_i(z^{(i-1)})] = z_i \qquad (12)$$

has a unique solution in the unit circle $|z_i| < 1$ for $|z_j| < 1$, $i + 1 \leq j \leq N$. Here $z^{(i-1)}$ denotes the vector z with the variables z_j replaced by σ_j for $1 \leq j \leq i - 1$. This unique solution is denoted by $\sigma_i(z_{i+1}, z_{i+2}, \cdots, z_N)$. The proof of this lemma is similar to the proof of Lemma 1.

If we let $p = 0$ and $z_N = 0$ in Lemmas 1 and 2 and we use the recursions defined by (8) and (12) for this case, then the unique functions $\hat{\sigma}_i(z_{i+1}, z_{i+2}, \cdots, z_{N-1})$, $1 \leq i \leq N - 2$, are defined, i.e., $\hat{\sigma}_1$ is the unique solution in the unit circle $|z_1| < 1$ of the equation $F(\hat{z}) Q_1(\hat{z}) = z_1$ where $\hat{z} = (z_1, z_2, \cdots, z_{N-1}, 0)$ given that $|z_i| < 1$, $2 \leq i \leq N - 1$. $\hat{\sigma}_i$, $2 \leq i \leq N - 2$, is the unique solution in the unit circle $|z_i| < 1$ of the equation $F(\hat{z}^{(i-1)}) Q(\hat{z}^{(i-1)}) = z_i$ where $\hat{z}^{(i-1)}$ is the vector $\hat{z}$ with $z_1 = \hat{\sigma}_1$, $z_2 = \hat{\sigma}_2, \cdots, z_{i-1} = \hat{\sigma}_{i-1}$ given that $|z_j| < 1$, $i + 1 \leq j \leq N - 2$. We are now armed enough to attack the problem of determining the $2N - 1$ boundary functions.

A. Determination of the Boundary Functions $\hat{G}_i(z)$, $0 \leq i \leq N - 2$

Letting $z_N \to 0$ in (6), we obtain

$$\hat{G}_0(z) = F(\hat{z})\Big\{ \hat{G}_{N-1}(z) + pQ_N(\hat{z}) G'_{N-1}(z)$$

$$+ \sum_{i=1}^{N-1} [\hat{G}_{i-1}(z) - \hat{G}_i(z)] z_i^{-1} Q_i(\hat{z}) \Big\} \qquad (13a)$$

where $\hat{z} = (z_1, z_2, \cdots, z_{N-1}, 0)$ and

$$G'_{N-1}(z) = \frac{dG_{N-1}(z)}{dz_N}\Big|_{z_N = 0}. \qquad (13b)$$

Notice that $G'_{N-1}(z)$ is a constant. Rearranging (13a) and noticing that by definition $\hat{G}_{N-1}(z) = G_N(z)$, we obtain

$$\hat{G}_0(z) = F(\hat{z}) \frac{E(\hat{z}) + \sum_{i=1}^{N-2} D_i(\hat{z}) \hat{G}_i(z)}{1 - F(\hat{z}) z_1^{-1} Q_1(\hat{z})} \qquad (14a)$$

where

$$E(\hat{z}) = [1 - z_{N-1}^{-1} Q_{N-1}(\hat{z})] \hat{G}_{N-1}(z) + pQ_N(\hat{z}) G'_{N-1}(z) \qquad (14b)$$

$$D_i(\hat{z}) = z_{i+1}^{-1} Q_{i+1}(\hat{z}) - z_i^{-1} Q_i(\hat{z}). \qquad (14c)$$

Notice that in (14) the boundary function $\hat{G}_0(z)$ is expressed in terms of the boundary functions $\hat{G}_i(z)$, $1 \leq i \leq N - 1$ and the constant $G'_{N-1}(z)$. Now, using the analytic property of $G_0(z)$, we immediately obtain the following result.

Theorem 2: Let $\hat{\sigma}_1$ and $\hat{z}^{(1)}$ be as defined before. Then,

$$\hat{G}_1(z) = F(\hat{z}^{(1)}) \frac{E(\hat{z}^{(1)}) + \sum_{i=2}^{N-2} D_i(\hat{z}^{(1)}) \hat{G}_i(z)}{1 - F(\hat{z}^{(1)}) z_2^{-1} Q_2(\hat{z}^{(1)})}. \qquad (15)$$

This is true since $\hat{G}_0(z)$ is an analytic function in the polydisk $|z_i| < 1$, $1 \leq i \leq N - 1$. Then, in this polydisk, whenever the denominator of $\hat{G}_0(z)$ vanishes, the numerator must also vanish. Since the denominator of $\hat{G}_0(z)$ vanishes at $\hat{\sigma}_1$, we have from (14) that

$$E(\hat{z}^{(1)}) + \sum_{i=2}^{N-2} [z_{i+1}^{-1} Q_{i+1}(\hat{z}^{(1)}) - z_i^{-1} Q_i(\hat{z}^{(1)})] \hat{G}_i(z)$$

$$= [\hat{\sigma}_1^{-1} Q_1(\hat{z}^{(1)}) - z_2^{-1} Q_2(\hat{z}^{(1)})] \hat{G}_1(z) \qquad (16)$$

which, together with the fact that $F(\hat{z}^{(1)}) \sigma_1^{-1} Q_1(\hat{z}^{(1)}) = 1$, implies (15).

Now, exploiting the similarity between (14) and (15) and repeating the above procedure for $i = 2, 3, \cdots, N - 2$, we obtain the following result.

Theorem 3: Let $\hat{\sigma}_i$ and $\hat{z}^{(i)}$, $2 \leq i \leq N - 2$, be as defined before. Then, for $\leq i \leq N - 2$,

$$\hat{G}_i(z) = F(\hat{z}^{(i)}) \frac{E(\hat{z}^{(i)}) + \sum_{j=i+1}^{N-2} D_j(\hat{z}^{(i)}) \hat{G}_j(z)}{1 - F(\hat{z}^{(i)}) z_{i+1}^{-1} Q_{i+1}(\hat{z}^{(i)})} \qquad (17)$$

The proof of (17) is similar to that of (15).

Now, using (17) for $i = N - 2$, we have

$$\hat{G}_{N-2}(z) = F(\hat{z}^{(N-2)}) \frac{E(\hat{z}^{(N-2)})}{1 - F(\hat{z}^{(N-2)}) z_{N-1}^{-1} Q_{N-1}(\hat{z}^{(N-2)})} \qquad (18)$$

and since $\hat{G}_{N-2}(z)$ is an analytic function for $|z_{N-1}| < 1$, we obtain from (18) and (14b) that

$$pG'_{N-1}(z) = \hat{G}_{N-1}(z) \frac{\sigma_{N-1}^{-1} Q_{N-1}(\hat{z}^{(N-1)}) - 1}{Q_N(\hat{z}^{(N-1)})}. \qquad (19)$$

Substituting (19) in (18), we get $\hat{G}_{N-2}(z)$ expressed in terms of the constant $\hat{G}_{N-1}(z)$. Using (17) for $i = N - 3, N - 4, \cdots, 2$, and then (15) and (14), we obtain all the functions $\hat{G}_i(z)$, $0 \leq i \leq N - 2$, expressed in terms of the constant $\hat{G}_{N-1}(z) = G_N(z)$. Specifically, as we shall need it later, let us define the function $k(\hat{z})$ as follows:

$$k(\hat{z}) = \hat{G}_0(z)/G_N(z). \qquad (20)$$

B. Determination of the Boundary Functions $G_i(z)$, $1 \le i \le N - 2$

To obtain the boundary functions $G_i(z)$, $1 \le i \le N - 2$, we use a procedure similar to that for $\hat{G}_i(z)$, $0 \le i \le N - 2$. Let us first rewrite (7a) as follows:

$$G(z) = F(z) \frac{H(z) + \sum_{i=1}^{N} H_i(z) G_i(z)}{1 - F(z)[p + \bar{p}z_1^{-1} Q_1(z)]} \quad (21a)$$

where $H_i(z)$, $1 \le i \le N$, are defined in (7b) and $H(z)$ is a known function up to the constant $G_N(z)$. $H(z)$ is given by

$$H(z) = \sum_{i=0}^{N-1} \hat{H}_i(z) \hat{G}_i(z). \quad (21b)$$

$\hat{H}_i(z)$ are defined in (7c).

Using Lemmas 1 and 2, we immediately obtain the following result.

Theorem 4: Let σ_i, $z^{(i)}$, $1 \le i \le N - 1$, be as defined in Lemmas 1 and 2. Then, for $1 \le i \le N - 2$, we have

$$G_i(z) = F(z^{(i)}) \frac{H(z^{(i)}) + \sum_{j=i+1}^{N} H_j(z^{(i)}) G_j(z)}{1 - F(z^{(i)})[p + \bar{p}z_{i+1}^{-1} Q_{i+1}(z^{(i)})]} \quad (22a)$$

and

$$G_{N-1}(z) = - \frac{H(z^{(N-1)}) + H_N(z^{(N-1)}) G_N(z)}{H_{N-1}(z^{(N-1)})}. \quad (22b)$$

We will demonstrate how (22a) is proved for $i = 1$. Then, by induction, one can easily obtain (22a) and (22b). Since $G(z)$ is an analytic function for $|z_i| < 1$, $1 \le i \le N$, and since the denominator of $G(z)$ vanishes at σ_1, we have from (21a) that

$$H(z^{(1)}) + \sum_{i=2}^{N} H_i(z^{(1)}) G_i(z) + H_1(z^{(1)}) G_1(z) = 0. \quad (23)$$

Using the definition of $H_1(z^{(1)})$ from (7b), i.e., $H_1(z^{(1)}) = \bar{p}[z_2^{-1} Q_2(z^{(1)}) - \sigma_1^{-1} Q_1(z^{(1)})]$, and the fact that $F(z^{(1)})[p + \bar{p}\sigma_1^{-1} Q_1(z^{(1)})] = 1$, we get immediately (22a) for $i = 1$.

Now, in (22b) $G_{N-1}(z)$ is expressed in terms of the constant $G_N(z)$. Using (22a) for $i = N - 2$, $N - 3$, $\cdots$, 1 we finally have all the boundary functions $G_i(z)$, $1 \le i \le N - 1$, expressed in terms of the constant $G_N(z)$.

Now that we have already determined $\hat{G}_i(z)$, $0 \le i \le N - 2$, and $G_i(z)$, $1 \le i \le N - 1$, in terms of the constant $G_N(z)$, the problem is reduced to that of determining this constant.

C. Determination of the Constant $G_N(z)$

To determine the constant $G_N(z)$, let us first prove the following.

Theorem 5: For $1 \le l \le N$, let

$$r_l = \frac{\partial F(z)}{\partial z_l}\bigg|_{z_1 = z_2 = \cdots = z_N = 1} \quad (24a)$$

and

$$\lambda_l = r_l + \sum_{j=1}^{N} \lambda_j \theta_j(l). \quad (24b)$$

Then the following holds:

$$\lambda_i = \bar{p}[G_{i-1}(1) - G_i(1)] + p[\hat{G}_{i-1}(1) - \hat{G}_i(1)]$$
$$1 \le i \le N - 1 \quad (25a)$$

$$\lambda_N = p[G_{N-1}(1) - G_N(1)] \quad (25b)$$

where

$$G_i(1) = G_i(z)\big|_{z_{i+1} = z_{i+2} = \cdots = z_N = 1} \quad 0 \le i \le N - 1 \quad (26a)$$

$$\hat{G}_i(1) = \hat{G}_i(z)\big|_{z_{i+1} = z_{i+2} = \cdots = z_{N-1} = 1} \quad 0 \le i \le N - 2 \quad (26b)$$

and $G_N(1) = \hat{G}_{N-1}(1)$ is just the constant we are looking for.

The proof of Theorem 5 appears in Appendix B. From (25), we obtain

$$\sum_{i=1}^{N-1} \lambda_i = \bar{p}[1 - G_{N-1}(1)] + p[\hat{G}_0(1) - \hat{G}_{N-1}(1)]$$
$$= \bar{p}[1 - \lambda_N/p - G_N(1)] + p[\hat{G}_0(1) - G_N(1)] \quad (27)$$

Therefore,

$$G_N(1) - p\hat{G}_0(1) = \bar{p}[1 - \lambda_N/p] - \sum_{i=1}^{N-1} \lambda_i. \quad (28)$$

Recalling that $\hat{G}_0(z) = k(\hat{z}) G_N(z)$, we finally have that

$$G_N(1) = \frac{\bar{p}[1 - \lambda_N/p] - \sum_{i=1}^{N-1} \lambda_i}{1 - pk(\hat{1})} \quad (29)$$

where $k(\hat{1}) = k(\hat{z})\big|_{z_1 = z_2 = \cdots = z_{N-1} = 1}$. Equation (31) implies that the condition for steady state is

$$\sum_{i=1}^{N-1} \lambda_i < \bar{p}(1 - \lambda_N/p). \quad (30)$$

Rewriting (30) as

$$\lambda_N < p\left(1 - \sum_{i=1}^{N-1} \lambda_i/\bar{p}\right) \quad (31)$$

we can explain the steady-state condition intuitively as follows. Clearly, node N is the bottleneck of the system.

If it is heavily loaded, then the fraction of time that the channel is used by the other $N - 1$ nodes is $\sum_{i=1}^{N-1} \lambda_i / \bar{p}$, so the fraction of time that the channel is available for node N for successful transmissions is $1 - \sum_{i=1}^{N-1} \lambda_i / \bar{p}$. As node N transmits with probability p when non-empty, the rate of its successful transmissions is $p(1 - \sum_{i=1}^{N-1} \lambda_i / \bar{p})$, which for stability must be greater than the total arrival rate to the node. Consequently, (31) should hold.

Having obtained the joint generating function $G(z)$ we can derive, at least in principle, any moment of the queue lengths at the nodes. Specifically, if we denote by L_i the average queue length at node i in steady state, then

$$L_i = \left. \frac{\partial G(z)}{\partial z_i} \right|_{z_1 = z_2 = \cdots = z_N = 1} . \tag{32}$$

Assuming that packets arrive at the nodes only at the end of a slot and then using Little's law [14], we may also obtain the average time delays at node i, denote by T_i, as follows:

$$T_i = L_i / \lambda_i \tag{33}$$

where λ_i is the total arrival rate at node i as defined in (24b). The total average time delay in the system is obtained by applying Little's law to the whole system, and it is given by

$$T = \sum_{i=1}^{N} L_i \Big/ \sum_{i=1}^{N} r_i \tag{34}$$

where r_i is the arrival rate at node i from its corresponding source as defined in (24a). The total average delay T is clearly a function of the transmission probability p. Obviously, as p decreases, the total average delay increases since node N transmits rather rarely. Also, when p increases, the total average delay also increases since there are many conflicts in the transmissions. Consequently, there is some intermediate value of p (that depends on the arrival processes to the nodes) that minimizes the total average delay in the system. This will be demonstrated in the example given in Section IV.

IV. EXAMPLE

In this section, we will use a simple example in order to show some details of the general solution method developed in the previous section. The example consists of a multihop network, depicted in Fig. 2, where packets arrive to nodes 1, 2, and 3, and node 2 forwards its packets to node 1. Consequently $Q_1(z) = Q_3(z) = 1$; $Q_2(z) = z_1$ (here $z = (z_1, z_2, z_3)$). We shall also assume that

$$F(z) = r_1 z_1 + r z_2 z_3 + 1 - r_1 - r$$

i.e., during each slot a packet arrives to node 1 with probability r_1, with probability r a packet arrives to both nodes 2 and 3, and with probability $1 - r_1 - r$ no packet arrives to the system. Then using (8), (12) for $z_3 = 0, p = 0$, we
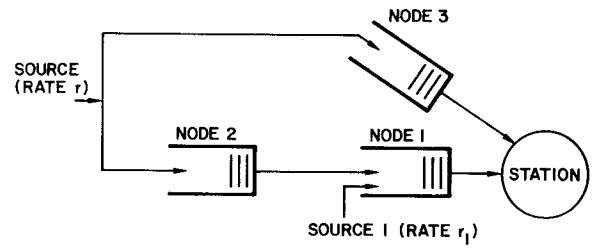

Fig. 2. Example network.

obtain

$$\hat{\sigma}_1 = 1 - \frac{r}{1 - r_1}$$

$$\hat{\sigma}_2 = \left(1 - \frac{r}{1 - r_1} \right)^2 .$$

Using (19), (18), and (14), we have

$$pG_2'(0, 0, 0) = \frac{r}{1 - r_1 - r} G(0, 0, 0)$$

$$\hat{G}(0, z_2, 0) = G(0, 0, 0)$$

$$\hat{G}(z_1, z_2, 0) = G(0, 0, 0) \left[1 + \frac{r_1}{1 - r_1 - r} z_1 \right] .$$

Using (29), we have that

$$G(0, 0, 0) = \frac{\bar{p}(1 - r/p) - (r_1 + 2r)}{1 - p(1 - r)/(1 - r_1 - r)}$$

and the condition for steady state is

$$\bar{p}(1 - r/p) - (r_1 + 2r) > 0.$$

From (8) and (11), we obtain

$$\sigma_1(z_2, z_3) = \left(1 - f(z_2, z_3) - r_1 \bar{p} - \sqrt{\Delta} \right) / 2r_1 p$$

where

$$f(z_2, z_3) = p(r z_2 z_3 + 1 - r_1 - r)$$

$$\Delta = \left(1 - f(z_2, z_3) - r_1 \bar{p} \right)^2 - 4r_1 \bar{p} f(z_2, z_3)$$

and $\sigma_2(z_3)$ is the solution of $\sigma_2(z_3) = \sigma_1^2(\sigma_2(z_3), z_3)$ in the unit circle $|\sigma_2| < 1$.

From (15) and (17), we obtain

$$G(0, 0, z_3) = G(0, 0, 0)$$

$$\cdot \frac{p(z_3^{-1} - 1) + \dfrac{r_1 p}{1 - r_1 - r} \left(1 - \sigma_1(\sigma_2(z_3), z_3) \right)}{1 - 2p + p z_3^{-1} - \bar{p} \sigma_1^{-1}(\sigma_2(z_3), z_3)}$$

and

$$G(0, z_2, z_3) = \left\{ G(0, 0, 0) \left[p(1 - z_3^{-1}) \right. \right.$$

$$\left. \left. + \frac{r_1 p}{1 - r_1 - r} \left(1 - \sigma_1(z_2, z_3) \right) \right] \right.$$

$$+ \ G(0, \ 0 \ z_3)\big(1 \ - \ 2p \ + pz_3^{-1}$$

$$- \ \bar{p}z_2^{-1}\sigma_1(z_2, \ z_3)\big)\bigg\} \Big/ \bigg\{ \ \bar{p}\big(\sigma_1^{-1}(z_2, \ z_3)$$

$$- \ z_2\sigma_1(z_2, \ z_3)\bigg\}$$

Finally, we have that

$$G(z_1, \ z_2, \ z_3) \ = \ F(z_1, \ z_2, \ z_3) \bigg\{ G(0, \ 0, \ 0) \ \bigg[p\big(1 \ - \ z_3^{-1}\big)$$

$$+ \ \frac{r_1 p}{1 \ - \ r_1 \ - \ r}\big(1 \ - \ z_1\big) \bigg] \ + \ G(0, \ 0, \ z_3)$$

$$\big(1 \ - \ 2p \ + pz_3^{-1} \ - \ \bar{p}z_2^{-1}z_1\big)$$

$$+ \ G(0, \ z_2, \ z_3) \ \bar{p}\big(z_2^{-1}z_1 \ - \ z_1^{-1}\big)\bigg\} \Big/ \big[1$$

$$- \ F(z_1, \ z_2, \ z_3)\big(p \ + \ \bar{p}z_1^{-1}\big)\big].$$

The explicit expressions for the average delays in this system are too complicated to be given here. To give some insight into the behavior of this network, we plotted these quantities in Figs. 3–5. In Fig. 3, T_1, T_2, T_3, and T are plotted as a function of $r = r_i$ for $p = 0.4$. In Fig. 4, these quantities are plotted as a function of p for $r_1 = r = 0.05$. As we can see, for small values of p, the queue is built up only at node 3 (since it is rarely transmitting), while for large values of p, queues are built up at all the nodes, and this is due to the interference.

As we see, there is an optimal transmission probability p^* that minimizes the total delay in the system. In Fig. 5, $T_{\min}$ (the minimal total delay in the system) is plotted as a function of $r = r_1$. It is interesting to mention that $p^* \approx 0.34$, and it is almost insensitive to the value of $r = r_1$. Also $T_{\min}$ is not very sensitive to small variations in p^*. As a final remark, we notice that when $r = r_1 \to 0$, the total average delay is given by $0.5 + 1/\bar{p} + 1/2p$, which is minimized when $p^* = 0.387$ and gets the value 3.423.

V. Summary and Discussion

In this paper, we studied a class of discrete-time priority queueing systems with partial interference. In these systems, N nodes share a common channel to transmit their packets. One node uses a random access scheme, while other nodes access the channel according to preassigned priorities. Packet arrivals are modeled as discrete-time batch processes, and packets are forwarded through the network according to fixed prescribed probabilities.

The motivation for considering these systems is twofold. First, their nontrivial analysis, along with the solution methodology that we develop for such systems, is of interest. Second, several conflict-free protocols for ac-
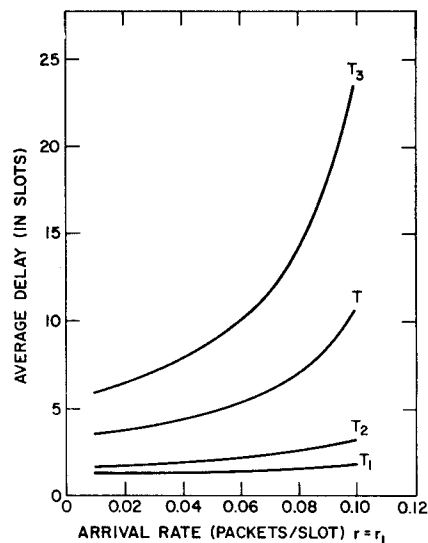

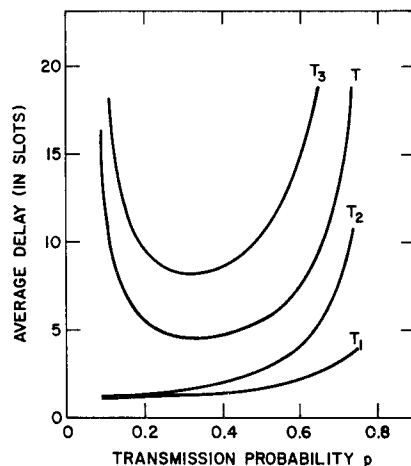Fig. 3. Average delays versus arrival rate.

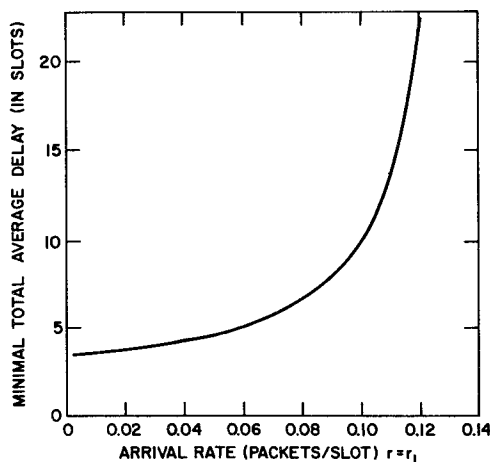Fig. 4. Average delays versus transmission probability.

Fig. 5. Minimal total average delay versus arrival rate.

cessing a common radio channel that have been suggested [6]–[8] can, with our model, be analyzed even in the presence of an interfering node. The restriction to a single interfering node is obviously due to the overwhelming

complexity in analyzing systems with multiple interfering nodes. Our method cannot be extended to deal with the case of more than a single interfering node, nor are we aware of any other existing method to analyze exactly systems with multiple interfering nodes. Evidently, the burden in developing any such method is that too many boundary functions need to be determined. The applicability of our results for a single interfering node in developing approximate solutions for systems with multiple interfering nodes, by aggregation for instance, is still to be explored.

In the paper, we provide steady-state analysis of the class of systems under consideration. In particular, we present a recursive method for the derivation of the joint generating function of the queue length distribution at the nodes in steady state. The condition for steady state is also derived.

APPENDIX A

Proof of Theorem 1: Consider the evolution equation (2) and let $G_t(z) = E\{\Pi_{i=1}^N z_i^{L_i(t)}\}$. Then,

$$G_{t+1}(z) = E\left\{\prod_{i=1}^N z_i^{L_i(t+1)}\right\} = F(z)$$
$$\cdot E\left\{\prod_{i=1}^N z_i^{L_i(t) + \Sigma_{m=1}^N D_{m(t)} - V_i(t)U_i(L_i(t))\Pi_{m=1}^{i-1}[1 - U_m(L_m(t))]}\right\}$$

(A1)

where in (A1) we used (1) and the fact that the vector of arrival processes $\{A_i(t)\}_{i=1}^N$ is independent of the state of the system.

Now, for $0 \le j \le N$, let the event that $L_i(t) = 0$ for $1 \le i \le j$ and $L_{j+1}(t) > 0$ be denoted by $\Omega_j(t)$. Then, from (A1) we obtain

$$G_{t+1}(z) = F(z)\left\{\Pr\left(\Omega_N(t)\right)\right.$$
$$+ \Pr\left(\Omega_{N-1}(t), V = 0\right)$$
$$\cdot E[z_N^{L_N(t)}/\Omega_{N-1}(t), V = 0]$$
$$+ \Pr\left(\Omega_{N-1}(t), V = 1\right)z_N^{-1}$$
$$\cdot E[z_N^{L_N(t)}/\Omega_{N-1}(t), V = 1]Q_N(z)$$
$$+ \sum_{j=0}^{N-2} \Pr\left(\Omega_j(t), L_N(t) = 0\right)z_j^{-1}$$
$$\cdot E\left[\prod_{m=j}^{N-1} z_m^{L_m(t)}/\Omega_j(t), L_N(t) = 0\right]Q_j(z)$$
$$+ \sum_{j=0}^{N-2} \Pr\left(\Omega_j(t), L_N(t) > 0, V = 0\right)z_j^{-1}$$
$$\cdot E\left[\prod_{m=j}^{N} z_m^{L_m(t)}/\Omega_j(t), L_N(t) > 0, V = 0\right]$$

$$\cdot Q_j(z) + \sum_{j=0}^{N-2} \Pr\left(\Omega_j(t), L_N(t) > 0, V = 1\right)$$
$$\cdot E\left[\prod_{m=j}^{N} z_m^{L_m(t)}/\Omega_j(t), L_N(t) > 0, V = 1\right]\right\}$$

(A2)

where in (A2), $z_0 \equiv 1$ and we used the definitions of $Q_j(z)$, $1 \le j \le N$. Now, since V is an independent random variable, we obtain from (A2)

$$G_{t+1}(z) = F(z)\left\{\Pr\left(\Omega_N(t)\right)\right.$$
$$+ \Pr\left(\Omega_{N-1}(t)\right) E[z_N^{L_N(t)}/\Omega_{N-1}(t)]$$
$$\cdot [\bar{p} + pz_N^{-1}Q_N(z)]$$
$$+ \sum_{j=0}^{N-2} \Pr\left(\Omega_j(t), L_N(t) = 0\right)$$
$$\cdot E\left[\prod_{m=j}^{N-1} z_m^{L_m(t)}/\Omega_j(t), L_N(t) = 0\right]z_j^{-1}Q_j(z)$$
$$+ \sum_{j=0}^{N-2} \Pr\left(\Omega_j(t), L_N(t) > 0\right)$$
$$\cdot E\left[\prod_{m=j}^{N} z_m^{L_m(t)}/\Omega_j(t), L_N(t) > 0\right]$$
$$\cdot [p + \bar{p}z_j^{-1}Q_j(z)]\right\}.$$

(A3)

Now, it is easy to see from (3) and (4) that for $t \to \infty$ we have

$$G_{t+1}(z) \to G(z)$$
$$\Pr\left(\Omega_N(t)\right) \to G_N(z)$$
$$\Pr\left(\Omega_{N-1}(t)\right) E[z_N^{L_N(t)}/\Omega_{N-1}] \to G_{N-1}(z) - G_N(z)$$
$$\Pr\left(\Omega_j(t), L_N(t) = 0\right) E\left[\prod_{m=j}^{N-1} z_m^{L_m(t)}/\Omega_j(t), L_N(t) = 0\right] \to \hat{G}_j(z) - \hat{G}_{j+1}(z)$$
$$\Pr\left(\Omega_j(t), L_N(t) > 0\right) E\left[\prod_{m=j}^{N} z_m^{L_m(t)}/\Omega_j(t), L_N(t) > 0\right] \to G_j(z) - G_{j+1}(z) - \hat{G}_j(z) + \hat{G}_{j+1}(z).$$

Therefore, (6) follows and Theorem 1 is proved □

APPENDIX B

Proof of Theorem 5: For $1 \le i \le N$, let us derive both sides of (6) with respect to z_i and substitute $z_1 = z_2 = \cdots = z_N = 1$. Then, for $1 \le i \le N - 1$, we obtain

$$0 = r_i + [G_{N-1}(1) - G_N(1)]p\theta_N(i)$$

$$+ \sum_{j=1}^{N-1} [\hat{G}_{j-1}(1) - \hat{G}_j(1)]\,\theta_j(i)$$

$$- [\hat{G}_{i-1}(1) - \hat{G}_i(1)]$$

$$+ \sum_{j=1}^{N-1} [G_{j-1}(1) - G_j(1) - \hat{G}_{j-1}(1)$$

$$+ \hat{G}_j(1)]\bar{p}\theta_j(i)$$

$$- \bar{p}[G_{i-1}(1) - G_i(1) - \hat{G}_{i-1}(1) + \hat{G}_i(1)]$$

and

$$0 = r_N - p[G_{N-1}(1) - G_N(1)]$$

$$+ \sum_{i=1}^{N-1} [\hat{G}_{i-1}(1) - \hat{G}_i(1)]\,\theta_i(N)$$

$$+ \sum_{i=1}^{N-1} [G_{i-1}(1) - G_i(1) - \hat{G}_{i-1}(1)$$

$$+ G_i(1)]\bar{p}\theta_i(N) \tag{B2}$$

where in (B1) and (B2) we used the fact that $G(1) = G_0(1) = 1$. Rearranging (B1) and (B2), we get for $1 \le i \le N - 1$:

$$0 = r_i + [G_{N-1}(1) - G_N(1)]p\theta_N(i)$$

$$+ \sum_{j=1}^{N-1} \left\{ \bar{p}[G_{j-1}(1) - G_j(1)] + p[\hat{G}_{j-1}(1) \right.$$

$$\left. - \hat{G}_j(1)] \right\} \theta_j(i)$$

$$- \left\{ \bar{p}[G_{i-1}(1) - G_i(1)] + p[\hat{G}_{i-1}(1) - G_i(1)] \right\} \tag{B3}$$

and

$$0 = r_N - p[G_{N-1}(1) - G_N(1)]$$

$$+ \sum_{i=1}^{N-1} \left\{ \bar{p}[G_{i-1}(1) - G_i(1)] \right.$$

$$\left. + p[\hat{G}_{i-1}(1) - \hat{G}_i(1)] \right\} \theta_i(N). \tag{B4}$$

In (B3) and (B4), we have N linear equations with N unknowns $\bar{p}[G_{i-1}(1) - G_i(1)] + p[\hat{G}_{i-1}(1) - \hat{G}_i(1)]$

for $1 \le i \le N - 1$ and $p[G_{N-1}(1) - G_N(1)]$. Apparently, (25) solves these equations. $\square$

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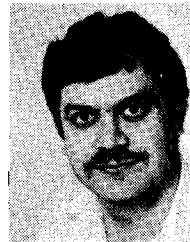
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