Erasure, Capture, and Noise Errors in Controlled Multiple-Access Networks

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Abstract—We consider an ALOHA-type communication system with many nodes accessing a common receiver through a time-slotted shared radio channel. Due to topological and environmental conditions, the receiver is prone to fail to detect some or all of the packets transmitted in a slot; this phenomenon is called *erasure*. The receiver may also *capture*, that is, detect a *single* transmission out of many. In addition, *noise errors* may cause the receiver to detect nonexistent collisions. Using the feedback information of detecting 0, 1 packet or a collision at the receiver, the nodes determine their transmission policy.

A class of decentralized multiaccess algorithms that maintain system stability under the above phenomena is presented, and the maximal throughput they can support is determined. The most remarkable feature of these algorithms is that their maximal throughput is insensitive to some noise errors and erasures.

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I. Introduction and Model Description

We consider ALOHA-type algorithms (ATA) that operate in presence of the following kind of errors [9]: 1) *noise errors*—no node is transmitting or a single node is transmitting, but the nodes are informed that there was a conflict in the channel; 2) *erasures*—at least one node is transmitting, but none of the transmitting modes are heard, hence the nodes are informed that the slot was idle; 3) *captures*—two or more nodes are transmitting simultaneously, but one of them captures the channel and is heard correctly, hence the nodes are informed that the slot contained a successful transmission. The effects of this kind of errors on collision resolution algorithms (CRA) have been studied in [5]–[9], and it was observed that CRA are quite sensitive to destructive errors such as noise errors and erasures. We study the effect of noise errors, erasures and captures on ATA. The ATA we use are those of [1]–[3] and we determine their stability region in the presence of noise errors, erasures and captures using the method of [2]. The remarkable property of these algorithms is their robustness in the presence of destructive errors.

We assume that new packets from all nodes are generated according to a Poisson process U_k with intensity λ packets/slot. The packet transmission duration is exactly one slot. At the end of each slot each node knows (without error, via some feedback channel) whether the slot was an idle slot, a success slot or a conflict slot. Moreover, if the slot is a success slot, the node whose packet was successful knows that (for instance, by recognizing its packet), so that a captured packet will not be retransmitted.

The model for errors is probabilistic. Specifically, when no node is transmitting, then the nodes detect an idle slot with probability π_{00} or a conflict (noise error) with probability $\pi_{0e} = 1 - \pi_{00}$. Similarly, when a single node is transmitting, the nodes detect an idle slot (erasure) with probability π_{10}, a success with probability π_{11} or a conflict (noise error) with probability π_{1e} ($\pi_{10} + \pi_{11} + \pi_{1e} = 1$). When two or more nodes are transmitting, the nodes detect an idle slot (erasure) with probability π_{e0}, a success (capture) with probability π_{e1} or a conflict with probability π_{ee} ($\pi_{e0} + \pi_{e1} + \pi_{ee} = 1$). When a conflict is detected, all packets (if any) transmitted during that slot have to be retransmitted at some later time. Similarly, when a success is detected, all packets (if any) that were transmitted during that slot, besides the successful packet, have to be retransmitted at some later time. Note that this model differs from that in [2]–[3] where the errors are assumed to affect *only the feedback information and not the actual transmissions.*

II. Stable ALOHA—Type Algorithms

The class of ALOHA-type algorithms presented here is essentially identical to that presented in [1]–[3]. Specifically, when a packet is generated at a node in slot j, then with probability f_{j+1} the packet is transmitted in slot $j + 1$. If the transmission is successful, then the packet leaves the network. Otherwise, (either the packet collided with other packets, or it was erased or corrupted by noise), the packet is retransmitted in subsequent slots, with probability f_k in slot k, until it is successfully transmitted. Note that a packet is successful only if it is either the only transmitted packet and it is neither erased nor corrupted by noise or if it is captured. The sequence $\{f_k\}$ is generated by

$$f_{k+1} = \min \{\beta, f_k a^{\gamma}(z_k)\} \tag{1}$$

where β is arbitrary $0 < \beta < 1$, $\gamma > 0$, and $a(\cdot)$ is a real-valued function. The feedback observed at the end of slot k, z_k, takes the values 0, 1, e according as an idle slot, a success slot or a conflict slot, respectively, are detected.

Let N_k be the number of packets waiting to be retransmitted at the beginning of slot k (the backlog). Then, when N_k is "large" or f_k is "small," the number of attempted transmissions in slot k is approximately Poisson with rate $\Lambda_k = N_k f_k$ [1], and the conditional throughput of the algorithm is

$$\text{Prob}\ \{\text{success}/N_k, f_k\} = \Lambda_k e^{-\Lambda_k}\pi_{11} + [1-(1+\Lambda_k)e^{-\Lambda_k}]\pi_{e1} \tag{2}$$

where the first term corresponds to transmission of a single packet that is neither erased nor corrupted by noise, and the second term corresponds to capture. For $\pi_{11} > \pi_{e1}$, the conditional throughput is maximized for $N_k f_k = \pi_{11}/(\pi_{11} - \pi_{e1})$ and its maximal value is

$$T^* = \pi_{e1} + (\pi_{11} - \pi_{e1})e^{-\pi_{11}/(\pi_{11}-\pi_{e1})}. \tag{3}$$

If $\pi_{11} \leq \pi_{e1}$ the maximal conditional throughput is π_{e1} (achieved with $f_k = 1$) and in this case the system behaves as a discrete-time queuing system with state-dependent service times. When $N_k \geq 2$, the service time is geometrically distributed with parameter π_{e1} and when $N_k = 1$ the service time is geometrically distributed with parameter π_{11}. Let $g_i = \lim_{k\to\infty} \text{Prob}\ \{N_k = i\}$, $i \geq 0$ and let $G(z) = \Sigma_{i=0}^{\infty} g_i z^i$. Then

$$G(z) = \frac{g_0(1-z^{-1})\pi_{e1} + g_1(1-z)(\pi_{11}-\pi_{e1})}{1-e^{\lambda(z-1)}[z^{-1}\pi_{e1}+1-\pi_{e1}]}\, e^{\lambda(z-1)} \quad \lambda < \pi_{e1} \tag{4}$$

where $g_0 = (\pi_{e1} - \lambda)/[\pi_{e1} + (e^{\lambda} - 1)(\pi_{e1} - \pi_{11})/\pi_{11}]$ and $g_1 = g_0(e^{\lambda} - 1)/\pi_{11}$. The system is stable for $\lambda < \pi_{e1}$ and the average number of backlogged users in steady-state is $E[N] = [2\pi_{e1}(1 - g_0) - \lambda^2]/[2(\pi_{e1} - \lambda)]$.

When $\pi_{11} > \pi_{e1}$ we follow the approach of [1]–[3]. Let $\phi_k = \ln\{\max(N_k, 1)f_k\}$, $\phi^* = \ln\{\pi_{11}/(\pi_{11} - \pi_{e1})\}$ and $m_k^* = \phi_k - \phi^*$. The $a(\cdot)$ should be chosen so that when the backlog grows, the product $N_k f_k$ drifts towards its optimal value, i.e., e^{ϕ^*}, thus maintaining the conditional throughput at its maximal value. To that end, m_k^* should drift towards zero. The expected conditional drift of m_k^* is $E[m_{k+1}^* - m_k^*/N_k, f_k] = E[\phi_{k+1} - \phi_k/N_k f_k]$ and when N_k is sufficiently large, it is approximately $E[\ln\{f_{k+1}/f_k\}/N_k, f_k]$, which for f_k small reduces to $\gamma E[\ln\{a(z_k)\}/N_k, f_k]$. Straightforward calculation of the last conditional expectation for large N_k yields

$$E[m_{k+1}^* - m_k^*/N_k, f_k] \approx \gamma d(\phi_k);$$
$$d(\phi_k) = \{e^{-G_k}, G e^{-G_k}, 1-(1+G_k)e^{-G_k}\} \cdot P\{C_o, C_1, C_2\}^T \tag{5}$$

where $G_k = e^{\phi_k}$; $C_o = \ln\{a\ (\text{idle})\}$, $C_1 = \ln\{a\ (\text{success})\}$, $C_2 = \ln\{(a\ (\text{conflict})\}$; $P = \{\pi_{ij}\}$ and i, j take the values 0, 1 and e; we assume throughout this correspondence that P is invertible. In calculating (5), binomial probabilities are approximated by Poisson probabilities, as in [1].

In order that $N_k f_k$ stays close to its optimal value, we need to choose C_o, C_1, C_2 so that $d(\phi_k) = 0$ for $\phi_k = \phi^*$, $d(\phi_k) < 0$ for $\phi_k > \phi^*$ and $d(\phi_k) > 0$ for $\phi_k < \phi^*$. With such a choice, when the backlog grows, the maximal achievable throughput increases to its maximal value T^*, so as long as $\lambda < T^*$ the system is stable. In the following we use the results of [1], [4] and a Liapunov function approach [2] to make this intuition precise and to show that a possible choice of C_o, C_1, C_2, is $\{C_o, C_1, C_2\}^T = P^{-1}\{e - 1, -1, -1\}^T$.

It may be desirable not to operate near ϕ^* (because of robustness considerations, see Section III). So, we fix some $\phi^o = \ln \Lambda^o$, which implies that f_k should satisfy $\phi^o \approx \ln\{\max(N_k, 1)f_k\}$. We show that if

$$\lambda < \Lambda^o e^{-\Lambda^o}\pi_{11} + [1-(1+\Lambda^o)e^{-\Lambda^o}]\pi_{e1} \tag{6}$$

then C can be chosen so that the system is stable for all γ small enough. To that end, define $m_k^o = \phi_k - \phi^o$, and assume that C is chosen so that

A1 $d(\phi^o) = 0$, and for some $\delta, \epsilon > 0$,

$d'(\phi) < 0$ for $|\phi - \phi^o| < \delta$;

$|d(\phi)| > \epsilon$ for $|\phi - \phi^o| \geq \delta$.

The choice of C is discussed in Section III, where we show that it can be chosen so the A1 holds.

To proceed, for $r, \Delta > 0$ define

$$g_\Delta(t) = \begin{cases} t^2 & |t| \leq \Delta \\ 2\Delta(t-\Delta/2) & t > \Delta \\ -2\Delta(t+\Delta/2) & t < -\Delta \end{cases} \tag{7}$$

$$V_k = N_k + r g_\Delta(m_k^o) \quad k = 0, 1, 2, \cdots. \tag{8}$$

If we show that for some $\alpha > 0$ and D, $E[e^{\alpha V_k}] < D$, for all k, then the stability is established, since $N_k \leq V_k$. To show that, we use a simplified version of [4, Theorem 2.3] (here $1(A) = 1$ if A is true, otherwise $1(A) = 0$).

Proposition: If for some $\alpha_1 > 0$ and D_1, $E[e^{\alpha_1 V_o}] \leq D_1$, and for some $s > 0$, $a > 0$

$$E[(V_{k+1} - V_k + s)1(V_k > a)/N_k, f_k] \leq 0 \tag{9}$$

$$|V_{k+1} - V_k| \leq \mu_o + \mu_a U_k \tag{10}$$

where μ_o, μ_a are constants and U_k are Poisson (λ), then for some $\alpha > 0$ and D, $E[e^{\alpha V_k}] \leq D$.

In the Appendix we prove that:

Lemma 1: Assume A1 and (6). Then there are some $s > 0$, $a > 0$ so that for γ small enough (9) holds.

Lemma 2: Equation (10) holds.

In the proof of Lemma 1 we essentially consider three cases; the case that N_k is large and m_k^o small ($\phi_k \approx \phi^o$); the case that N_k is large and m_k^o large ($\phi_k \neq \phi^0$); the case that N_k is small. When N_k is large and m_k^o is small, then on the average $g_\Delta(m_k^o)$ does not change much, so (6) implies (9). If $|m_k^o|$ is large, then on the average $g_\Delta(m_k^o)$ is decreasing, so choosing r large ensures that V_k will decrease on the average. If N_k is small and V_k is large, then necessarily f_k is smaller than it's optimal value, so g_Δ will decrease. In the proof we indicate the values for the constants r, Δ, a and γ_o so that for all $\gamma < \gamma_o$, (9) holds. Lemma 2 holds since g_Δ has linear growth, and changes in N_k are Poisson.

Finally, using the proposition, Lemma 1 and 2, and the fact that $0 \leq N_k \leq V_k$, we obtain the following.

Stability Theorem: Assume A1 and (6). Assume N_o and f_o are such that, for some $\alpha_1 > 0$ and D_1, $E[e^{\alpha_1 V_o}] < D_1$. Then for sufficiently small γ and for some $\alpha > 0$, $E[e^{\alpha V_k}] \leq D < \infty$ and so the system is stable.

III. Sensitivity and Robustness

Assuming that the matrix P is nonsingular and that $\pi_{e1} < \pi_{11}$, it is straightforward to show that the choice $\{C_o, C_1, C_2\}^T = P^{-1}\{e - 1, -1, -1\}^T$ satisfies condition A1 for $\phi^0 = \phi^*$, and therefore ensures that the system is stable for $\lambda < T^*$. Let $\{\tilde{C}_o, \tilde{C}_1, \tilde{C}_2\}^T = P\{C_o, C_1, C_2\}^T$, then it is easy to see that if $\tilde{C}_2 < 0 < \tilde{C}_0$, then for some ϕ^o A1 holds.

An important feature of the algorithm presented here is that the maximal throughput it can support depends solely on the probabilities of successful transmissions π_{11} and π_{e1} and therefore it is not sensitive at all to destructive errors such as noisy idle slot or erasure of at least two packets (unlike CRA). Thus, as long as P is invertible, the system will be stable for any arrival rate λ that is smaller than the maximal throughput. This insensitivity property is due to the ability to easily incorporate the effects of the destructive errors into the way one computes the retransmission probabilities (by a proper choice of C).

Another nice property of the algorithm is that there are some degrees of freedom in choosing C_0, C_1, and C_2. Consequently, one can choose these parameters so that the operation of the algorithm (the way the retransmission probabilities are updated) does not depend on some of the probabilities of erasure, capture, or noise. Consider the case where only π_{1e} and π_{e1} are positive: then $C_1 = C_2$ is a feasible choice and the operation of the algorithm does not depend on π_{1e} and π_{e1}. Consequently, the algorithm can be applied without even knowing π_{1e} and π_{e1}! The maximal throughput of the algorithm does depend on these probabilities [see (3)].

The algorithm is also robust to changes in the error probabilities in the following sense. Assume that $\pi_{e1} < \lambda < T^*$ (for $\lambda < \pi_{e1}$, taking $f_k = 1$ yields a stable algorithm for any values of the destructive errors). For such λ the equation $\lambda = \Lambda e^{-\Lambda}\pi_{11} + [1 - (1 + \Lambda)e^{-\Lambda}]\pi_{e1}$ has two distinct solutions in the range $\Lambda \geq 0$. Let Λ^1 and Λ^2 denote the solutions where $\Lambda^1 < \Lambda^2$ (Note that $\Lambda^1 < \pi_{11}/(\pi_{11} - \pi_{e1}) < \Lambda^2$). Fix some $\Lambda^0(\Lambda^1 < \Lambda^0 < \Lambda^2)$, and let $\phi^0 = \ln \Lambda^0$. Choose some C so that A1 holds. Now assume that the system operates with this set of C and some of the error probabilities such as π_{e0} or π_{0e}, etc., are changed. This may reflect imperfect knowledge of the system parameters. Due to the continuity of $d(\phi^0)$ in all parameters, the system would remain stable for any slight changes in the parameters (even though C is not changed). Moreover, for a proper choice of C, even relatively large variations in some of the system parameters would still yield a stable system [12]. Consequently, the algorithm is quite robust to imperfect knowledge of the parameters of the system.

Appendix

Denote by e_f, e_N, and e_{Nf} some functions which are bounded by Kf, KN^{-1} or both, respectively; these change from usage to usage. $q_i(\tilde{q}_i)$ denote the probabilities $P(X = i)$ for Binomial (f, n) [Poisson (fn) resp.], $i = 0, 1$, $q_2 := 1 - q_0 - q_1$ and similarly for $\tilde{q}$. The Poisson approximation [1], [12] gives

$$|q_i - \tilde{q}_i| \leq e_{nf}, \quad i = 0, 1, 2. \tag{A1}$$

Details of this and claims that follow are in [12]. In addition, we have the following.

Lemma A: 1) $|\phi_{k+1} - \phi_k| \leq [(N_k - 1) \vee 1]^{-1}(U_k + 1) + \gamma K$ where U_k is Poisson (λ). Also

$$2)\ E((\phi_{k+1} - \phi_k)^2 | N_k, f_k) \leq K[(N_k - 1) \vee 1]^{-2} + K\gamma^2. \tag{A2}$$

Proof: $|\phi_{k+1} - \phi_k| \leq |\ln (N_{k+1} \vee 1/N_k \vee 1)| + |\ln ([f_k a^\gamma(z_k)] \wedge \beta)/f_k| \leq 1/((N_k - 1) \vee 1)|N_{k+1} - N_k| + \max_i \gamma |\ln a(i)|$, and 1) follows. Using this calculation (A2) follows since U_k is Poisson and independent of $N_k f_k$.

Proof of Lemma 1: We use Poisson approximation. Define $\lambda_k = N_k f_k$. Using (A1),

$$E(N_{k+1} - N_k | N_k, f_k) = \lambda - \{\lambda_k e^{-\lambda_k}\pi_{11} + [1 - (1 + \lambda_k)e^{-\lambda_k}]\pi_{e1}\} + e_{Nf}. \tag{A3}$$

Denote $\bar{f} = \beta/\max_i |a(i)|$ and note that $f_k \leq \bar{f}$ implies $f_{k+1} = f_k \cdot a^\gamma(z_k)$. We get

$$E(\phi_{k+1} - \phi_k | N_k, f_k) = E\left[\ln \frac{N_{k+1} \vee 1}{N_k \vee 1} + \ln \frac{[f_k a^\gamma(z_k)] \wedge \beta}{f_k}\right]$$
$$= E\left[\ln \frac{N_{k+1} \vee 1}{N_k \vee 1}\right] + \gamma E(\ln a(z_k)) + e_1(f_k) \tag{A4}$$

where, since $\ln(\cdot)$ in increasing, $e_1(f) \leq 0$ and equals 0 if $f \leq \bar{f}$. Using (A1) and Lemma A,

$$E(\phi_{k+1} - \phi_k | N_k, f_k) = e_N + \gamma\tilde{q}(\lambda_k)PC + \gamma e_{Nf} + e_1(f_k). \tag{A5}$$

Since by Taylor's theorem $g_\Delta(x) - g_\Delta(y) = g'_\Delta(y)(x - y) + \alpha(x, y)(x - y)^2$ with $|\alpha| \leq 1$,

$$E(rg_\Delta(m_{k+1}) - rg_\Delta(m_k) | N_k, f_k) = rg'_\Delta(m_k)E(\phi_{k+1} - \phi_k | N_k, f_k) + rE(\alpha(m_k, m_{k+1})(\phi_{k+1} - \phi_k)^2 | N_k, f_k). \tag{A6}$$

By (A5) the first term equals $rg'_\Delta(m_k)\gamma\tilde{q}(\lambda_k)PC + rg'_\Delta(m_k)[e_N + \gamma e_{Nf} + e(f_k)]$ where $|g'_\Delta| \leq 2\Delta$. Since $|\alpha| \leq 1$, Lemma A implies that the second term in (A6) is bounded by $r\gamma^2 K + rK((N_k + 1) \vee 1)^{-2}$. Collecting terms and using (A3), (A6) we obtain the approximation for the dynamics V_k

$$\begin{aligned} E(V_{k+1} - V_k | N_k, f_k) &= \lambda - \{\lambda_k e^{-\lambda_k}\pi_{11} + [1 - (1 + \lambda_k)e^{-\lambda_k}]\pi_{e1}\} \\ &\quad + rg'_\Delta(m_k)\gamma\tilde{q}(\lambda_k)PC \\ &\quad + e_{Nf} + rg'_\Delta(m_k)[e_N + \gamma e_{Nf} + e_1(f_k)] \\ &\quad + r\gamma^2 K + rK((N_k + 1) \vee 1)^{-2}. \end{aligned} \tag{A7}$$

If $V_k > a$, then either $N_k > a_1$ or $rg_\Delta(m_k) > a_2$ where $a_1 + a_2 = a$. If N_k is large and m_k is small, choose x so that $|\psi - \phi^0| < x \leq \delta$ implies [c.f. (2)] that for some positive s, $e^\psi e^{-e^\psi}\pi_{11} + (1 - (1 + e^\psi)e^{-e^\psi})\pi_{e1} - \lambda > 2s$, that is, the (approximate) probability of success is larger than the arrival rate; this is possible by (7). Note that by A1 and (7), $g'_\Delta(m_k)d(\phi_k) := g'_\Delta(m_k)\tilde{q}(\lambda_k)PC \leq 0$, and that if N_k is large and m_k small, then necessarily f_k is small, and so $e_1(f_k) = 0$. Choose a_3 large and γ_1 small enough so that $N_k > a_3$, $\gamma < \gamma_1$ imply

$$e_{Nf} + r2\Delta[e_N + \gamma e_{Nf}] + r\gamma^2 K + rKN_k^{-2} < s. \tag{A8}$$

Then, for $N_k > a_3$ and $|\phi_k - \phi^0| < x$, (9) holds. Now suppose $N_k > a_3$ is large and m_k small, so $|\psi - \phi^0| > x$. Note that the second term in (A7) is uniformly bounded in λ_k and that, if $\Delta > x$ and $|\phi_k - \phi^o| > x$, then by A1 for some $\epsilon_1 > 0$, $g'_\Delta(m_k)\tilde{q}(\lambda_k)PC \leq -2x\epsilon_1$. Now either $\phi_k < \phi^o$, so $f_k < \bar{f}$ so $e_1(f_k) = 0$, or, if $\phi_k > \phi^o$, then $g'_\Delta(m_k)e_1(f_k) \leq 0$. So, choosing r_o large, for $r = r_o/\gamma$, $\lambda - 2x\epsilon_1\gamma r + 2s < 0$. Now choose $\gamma_2 < \gamma_1$ small and for any $\gamma < \gamma_2$, choose $a_4 = a_4(\gamma) > a_3$ so that this and (A8) both hold. Then (9) holds for $N_k > a_4$ and all ϕ_k. Finally for $N_k < a_4$ and $rg_\Delta(\phi_k) > a_2$ large, necessarily f_k is small. Choose $b > 1$ and a_2 so large that

$$f_k < \bar{f} \text{ so } e_1(f_k) = 0, \ |\phi_k - \phi_0| > b \cdot \Delta,$$

$$E\left[\ln \frac{N_{k+1} \vee 1}{N_k \vee 1} \middle| N_k, f_k\right] \geq 0. \tag{A9}$$

This is possible whenever $\lambda > 0$. Recalling (A5), (A9) we have $E[\phi_{k+1} - \phi_k | N_k, {}_k] \geq \gamma\tilde{q}PC + \gamma e_{Nf}$. From the definition of g_Δ, under (A9) this function is affine at ϕ_k with slope (-2Δ), so

$$E[g_\Delta(m_{k+1}) - g_\Delta(m_k) | N_k, f_k] \leq -2\Delta E[\phi_{k+1} - \phi_k | N_k, f_k] + E[|g_\Delta(m_{k+1}) - (-2\Delta(m_{k+1} + \Delta/2))| \, | N_k, f_k] \tag{A10}$$

where the last term is 0 if $m_{k+1} < -\Delta$. We show that by choosing b large enough, the last term can be ignored. Since $|g_\Delta(m_{k+1})| \leq |m_{k+1}|$, we have $e_3 := |g_\Delta(m_{k+1}) + 2\Delta(m_{k+1})$

$+ \Delta/2)| \leq 4\Delta|m_{k+1}| + \Delta^2$ and

$$|m_{k+1}| \leq \ln (N_k + U_k + 1) + |\ln f_k a^\gamma(z_k)| + |\phi^o|$$
$$\leq \ln a_4 + \ln (1 + U_k/a_4) + \gamma K + |\phi^o|$$
$$\leq K + \frac{U_k}{a_4}. \quad \text{(A11)}$$

But $m_{k+1} > -\Delta$ and $m_k < -b\Delta$ implies $U_k \geq e_4(b)$ where $e_4(b)$ increases with b (Lemma A). Thus,

$$E[e_3|N_k, f_k] \leq E([K + KU_k]1[U_k > e_4(b)]|N_k, f_k)$$
$$= E([K + KU_k]1[U_k > e_4(b)]) \quad \text{(A12)}$$

since U_k is independent of N_k, f_k. The last term goes to zero as $b \to \infty$, and so does the last term of (A10). Using this and A1 we get for large b and some $a_5 > a_2$ (so that f_k is small enough)

$$E[g_\Delta(\phi_{k+1}) - g_\Delta(\phi_k)|N_k, f_k] \leq -2\Delta\gamma\tilde{q}PC$$
$$+ 2\Delta\gamma e_{Nf} + E(e_3(b)|N_k, f_k) \leq -\Delta\gamma\epsilon. \quad \text{(A13)}$$

Now $E(V_{k+1} - V_k|N_k, f_k) = \lambda - \{\lambda_k e^{-\lambda_k}\pi_{11} + [1 - (1 + \lambda_k)e^{-\lambda_k}\pi_{e1}\} + e_{Nf} - r\gamma\Delta\epsilon \leq -s$ (A3), (A13) if $r_o = r \cdot \gamma$ is large and f_k small, since e_{Nf} is bounded, so (9) holds for $a = a_2 + a_4$ and γ small enough.

Proof of Lemma 2: $|V_{k-1} - V_k| \leq |N_{k+1} - N_k| + r|g_\Delta(m_{k+1}) - g_\Delta(m_k)| \leq |N_{k+1} - N_k| + 2r\Delta|\phi_{k+1} - \phi_k|$. But from the proof of Lemma A, the last term is smaller then $2r\Delta|N_{k+1} - N_k| + rK$, and so $|V_{k+1} - V_k| \leq 2|N_{k+1} - N_k| + rK = \mu_o + \mu_a U_k$ and the Lemma is established.

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