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An Acknowledgment-Based Access Scheme in a Two-Node Packet-Radio Network

MOSHE SIDI AND ADRIAN SEGALL

Abstract—**A two-node packet-radio network with *infinite buffers* at the nodes is considered. The two nodes transmit data packets to a common station through a shared radio channel. One of the two nodes is granted full access rights to the channel, while the other node bases its decisions whether to transmit or not on the acknowledgments it receives about its transmissions. For this acknowledgment-based access scheme and for general arrival processes, we derive the joint generating function of the queue contents in steady state, as well as the condition for steady state. From the generating function, any moment can be derived, as well as average time delays. Numerical results are presented for independent Bernoulli arrival processes.**

I. INTRODUCTION

In packet-radio networks, many contending devices share a common radio channel in a given locality. It has been observed [1], [5]-[7] that within such an environment, the outcome of the transmission of a packet by a node depends on both the states and actions of neighboring nodes. This dependence inhibits any attempt to obtain explicit analytic results for general networks and access schemes.

As an initial attempt to understand the behavior of packet-radio networks, there is a need to first accurately analyze simple yet typical configurations. In this paper, we consider the two-node packet radio network depicted in Fig. 1. In this network, the two nodes having *infinite buffers* send data packets arriving from outside sources to a common station over a shared radio channel. The channel is assumed to be divided into slots whose durations correspond to the transmission time of a packet, and a node may start transmission of a packet only at the beginning of a slot.

In [1], we analyzed the behavior of this network for a certain random access scheme. We assumed in [1] that one of the nodes (node 2) is granted full access rights to the channel, i.e., it transmits whenever its queue is nonempty. The access rights of the other node (node 1) are randomized as follows. At the beginning of each slot for which its own buffer is nonempty, node 1 tosses a coin with probability of success p, independently of any other event in the system, and in case of success, it attempts to transmit the packet at the head of its queue.

In the present paper, we analyze a different access scheme

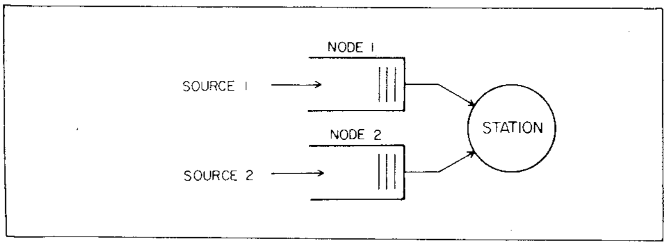

Fig. 1. A two-node packet-radio network.

applied to the same network. We still assume that node 2 transmits whenever its queue is nonempty. However, we assume that the decision at node 1 whether or not to transmit in a given slot depends on what has happened in the previous slot. Specifically, if a collision occurred in the previous slot, the nodes know for sure that they both have packets ready for transmission. Since node 2 is always allowed to transmit, it makes sense that node 1 should refrain from transmitting in the current slot in order to ensure successful transmission from node 2. In the following section, we shall present the access scheme at node 1 in detail.

For this acknowledgment-based access scheme, we obtain the condition for steady state and the generating function of the steady-state joint probability distribution of the queue lengths at the two nodes. From this generating function, any moment can be derived, as well as average time delays. We also give some numerical results for independent Bernoulli arrival processes, and we compare the acknowledgment-based access scheme to the random access scheme of [1].

II. THE MODEL

Packets arrive randomly at the two nodes (Fig. 1) from the outside of the system, and in general, the arrival processes may be correlated. Let $A_1(t)$ and $A_2(t)$ be the number of packets entering nodes 1 and 2 from their corresponding sources in the time interval $(t, t+1]$. The input process $[A_1(t), A_2(t)]$ is assumed to be a sequence of independent and identically distributed random vectors with integer-valued elements. Let

$$F(x, y) = E[x^{A_1(t)} y^{A_2(t)}]. \tag{1}$$

We assume that $F(x, y)$ depends on both x and y, namely, that packets arrive at the two nodes with nonzero probability and that the two nodes have infinite buffers. The packets at the nodes are transmitted to the main station over a common radio channel.

It is assumed that both nodes receive instantaneous acknowledgment at the end of each slot, whether or not their transmission was successful. Unsuccessful transmissions are due only to collisions. Collided packets remain at the head of the queue (at each node), and the nodes try to transmit them again according to the schemes that we now describe.

As mentioned earlier, node 2 attempts transmission whenever its queue is nonempty. The action of node 1 in a given slot depends on the channel activity during the previous slot. If a collision (simultaneous transmissions) has occurred in the previous slot, node 1 remains silent. This is because it knows that node 2 will attempt to transmit its packet again and it can gain no advantage by trying to transmit. If there was no colli-

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M. Sidi is with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa, Israel, on leave at the Laboratory for Information and Decision Systems, Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139.

A. Segall is with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa, Israel, on leave at the IBM T. J. Watson Research Center, Yorktown Heights, NY 10598.

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sion in the previous slot, node 1 tosses a coin with probability of success p. In case of success, it attempts to transmit the packet at the head of its queue; otherwise, it remains silent.

III. Steady-State Distribution

In this section, we shall analyze the steady-state behavior of the acknowledgment-based access scheme described above. We say that the network is in state $S = 1$, in a given slot, if there was no collision in the previous slot, and in state $S = 2$ otherwise. Let $P_k(m, n)$, $k = 1, 2$, $m \geqslant 0$, $n \geqslant 0$ be the equilibrium joint probability that the system is in state k and the queue lengths at nodes 1 and 2 are m and n, respectively. Let

$$G_k(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P_k(m, n) x^m y^n, \qquad k = 1, 2 \tag{2}$$

be the queue length joint generating function when the system is in state k ($k = 1, 2$). Then, using a standard technique, it can be shown that (here $\bar{p} = 1 - p$)

$$\begin{aligned} G_1(x, y) = F(x, y)\{&G_1(0, 0) + [G_1(0, y) - G_1(0, 0)]y^{-1} \\ &+ [G_1(x, 0) - G_1(0, 0)](px^{-1} + \bar{p}) \\ &+ G_2(x, y)y^{-1} + [G_1(x, y) - G_1(x, 0) \\ &- G_1(0, y) + G_1(0, 0)]\bar{p}y^{-1}\} \end{aligned} \tag{3a}$$

$$\begin{aligned} G_2(x, y) = F(x, y)p[&G_1(x, y) - G_1(x, 0) \\ &- G_1(0, y) + G_1(0, 0)]. \end{aligned} \tag{3b}$$

In (3), we encounter the common phenomenon in coupled queues, that the generating functions $G_k(x, y)$, $k = 1, 2$, are expressed in terms of the boundary functions $G_1(0, y)$, $G_1(x, 0)$ and the constant $G_1(0, 0)$. In order to determine $G_k(x, y)$, $k = 1, 2$, uniquely, we still have to determine these boundary functions and this constant.

Determination of $G_1(0, y)$

To determine $G_1(0, y)$, let $x \to 0$ in (3a). Noticing from (3b) that $G_2(0, y) = 0$, we obtain

$$G_1(0, y) = F(0, y) \frac{G_1(0, 0)(1 - y^{-1}) + pP_1(1, 0)}{1 - F(0, y)y^{-1}}. \tag{4}$$

Applying Rouche's theorem [3], it is easy to see that the equation $F(0, y) = y$ has a unique solution in the unit circle $|y| < 1$. Let this solution be denoted by σ. Then, since $G_1(0, y)$ is analytic for $|y| < 1$, we obtain from (4) that $pP_1(1, 0) = G_1(0, 0)(\sigma^{-1} - 1)$. Therefore,

$$G_1(0, y) = F(0, y) \frac{(\sigma^{-1} - y^{-1})G_1(0, 0)}{1 - F(0, y)y^{-1}} \tag{5}$$

and $G_1(0, y)$ is determined up to the constant $G_1(0, 0)$.

Determination of $G_1(x, 0)$

From (3), we obtain

$$G_1(x, y) = F(x, y) \cdot \frac{b(x, y)G_1(x, 0) + c(x, y)G_1(0, y) + d(x, y)G_1(0, 0)}{xe(x, y)} \tag{6a}$$

where

$$b(x, y) = y(p + \bar{p}x) - x[\bar{p} + pF(x, y)]; \tag{6b}$$

$$c(x, y) = xp[1 - F(x, y)]; \tag{6c}$$

$$d(x, y) = pxy - x - py + x[\bar{p} + pF(x, y)]; \tag{6d}$$

$$e(x, y) = y - F(x, y)[\bar{p} + pF(x, y)]. \tag{6e}$$

Now again applying Rouche's theorem, it can be shown that for given x, $|x| < 1$, the equation (in y) $e(x, y) = 0$ has a unique solution in the unit circle $|y| < 1$. Let this solution be denoted by $f(x)$. Since $G_1(x, y)$ is analytic in the polydisk $|x| < 1$, $|y| < 1$, we immediately obtain from (6a) that

$$G_1(x, 0) = -\frac{c(x, f(x))G_1(0, f(x)) + d(x, f(x))G_1(0, 0)}{b(x, f(x))}. \tag{7}$$

Substituting (5) in (7) determines $G_1(x, 0)$ up to the constant $G_1(0, 0)$.

Finally, $G_1(0, 0)$ is determined via the normalization condition $G_1(1, 1) + G_2(1, 1) = 1$. After tedious calculation, we obtain

$$G_1(0, 0) = \frac{1 - r_2(1 + p) - r_1/p}{1 + p \dfrac{\sigma - F(0, 1)}{\sigma[1 - F(0, 1)]}} \tag{8}$$

where r_1 and r_2 are the arrival rates into nodes 1 and 2, respectively, and are given by

$$r_1 = \left.\frac{\partial F(x, y)}{\partial x}\right|_{x=y=1}; \quad r_2 = \left.\frac{\partial F(x, y)}{\partial y}\right|_{x=y=1}. \tag{9}$$

The condition for steady state is $G_1(0, 0) > 0$.

Thus, we have completely determined the generating functions $G_k(x, y)$, $k = 1, 2$, and in principle, any moment of the queue lengths at the nodes as well as average time delays (using Little's law [4]) can be derived.

IV. Independent Bernoulli Arrival Processes

Although our results in the previous section were derived for general arrival processes, some of the expressions are simplified when we consider independent Bernoulli arrival processes into the nodes, i.e., $F(x, y) = (xr_1 + \bar{r}_1)(yr_2 + \bar{r}_2)$ where r_1, r_2 are the arrival rates into nodes 1 and 2, respectively, and $\bar{r}_i = 1 - r_i$ for $i = 1, 2$.

From (8), we immediately obtain that, in this case,

$$G_1(0, 0) = \frac{\bar{r}_2[1 - r_2(1 + p) - r_1/p]}{1 - r_2(1 + p)} \tag{10}$$

and the condition for steady state is that $1 - r_2(1 + p) - r_1/p > 0$.

The explicit expressions for the average queue lengths at the nodes, and therefore for the average delays, are too complicated to be given here. However, we shall present several graphs that exhibit the behavior of the latter quantities. In Figs. 2 and 3, the average time delays T_1 and T_2, at nodes 1 and 2 respectively, are plotted versus p, the transmission probability at node 1, and in Fig. 4, the total average delay in the network-T is plotted versus p for $r_1 = 0.1$, and r_2 ranges from 0.01 to 0.5. From Fig. 4, we see that for given r_1, when r_2 is small, the optimal p (that minimizes the total average delay) is 1, i.e., node 1 has to transmit a packet whenever it has any, except that after a collision, it should remain silent for one slot. On the other hand, when r_2 increases, the optimal p decreases since for large values of p, the probability of a collision is large. It is also interesting to mention that when

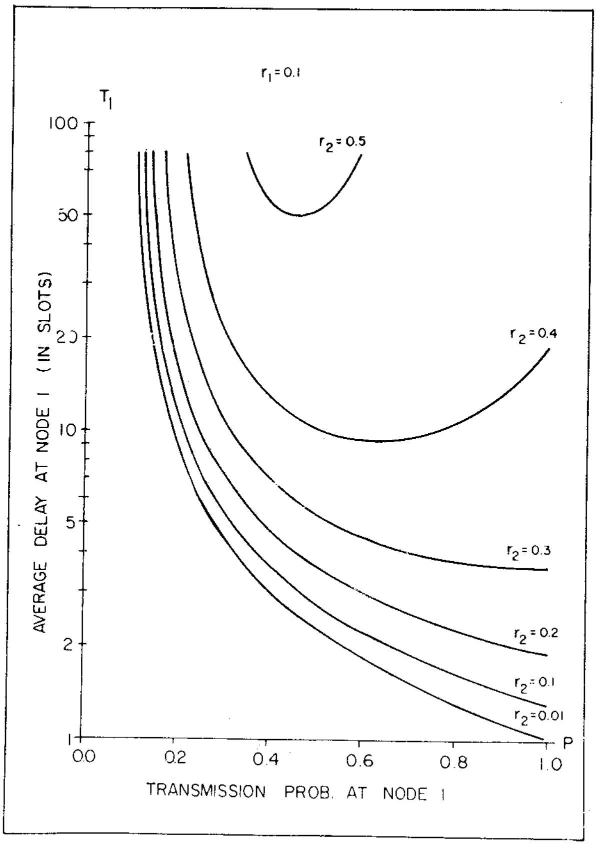

Fig. 2. Average delay at node 1 versus the transmission probability at node 1.

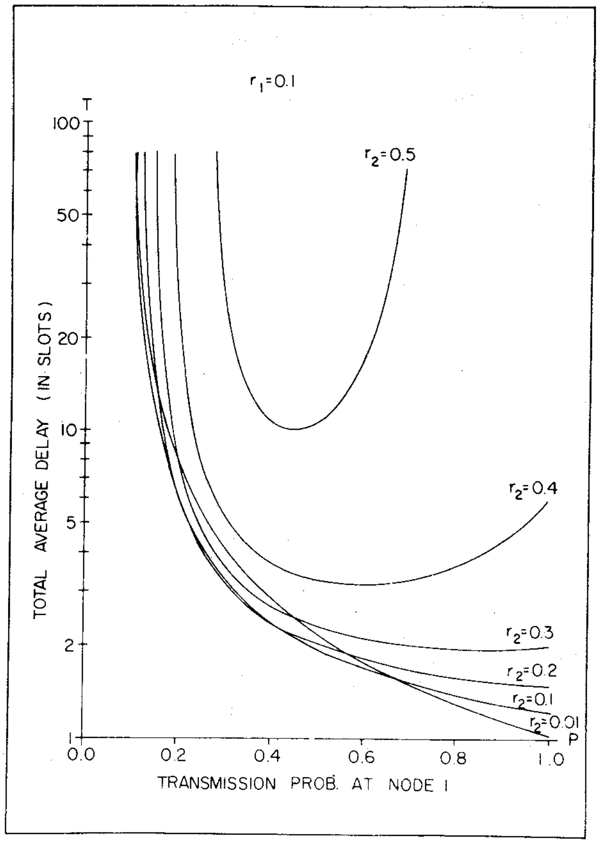

Fig. 4. Total average delay versus the transmission probability at node 1.

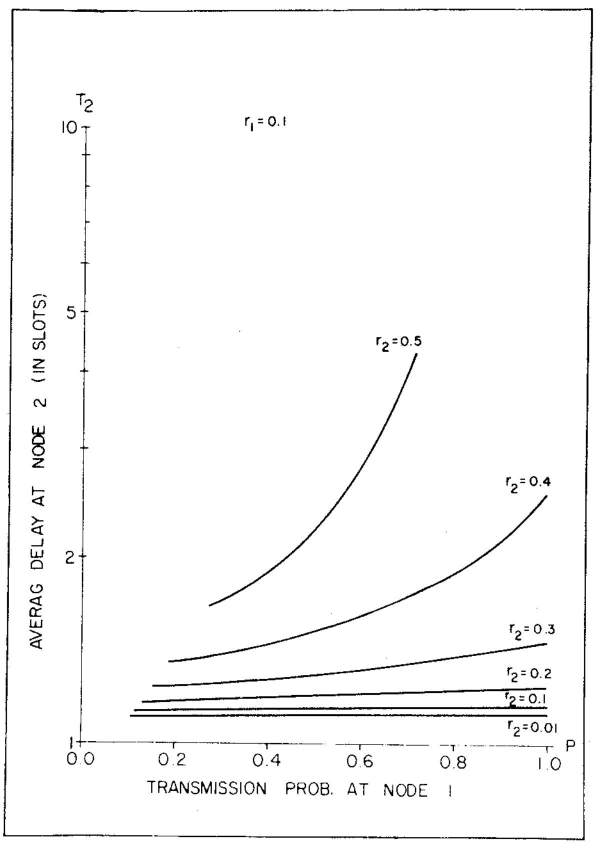

Fig. 3. Average delay at node 2 versus the transmission probability at node 1.

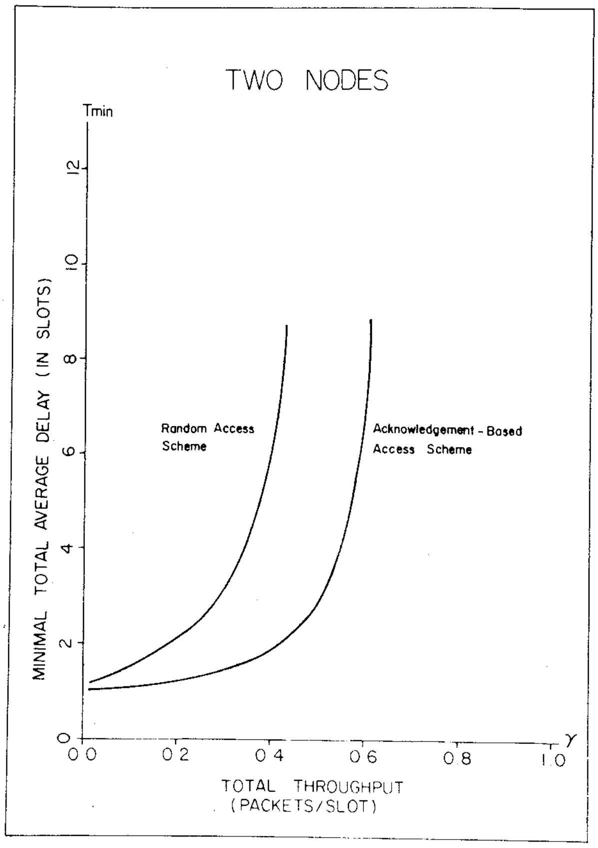

Fig. 5. Minimal total average delay versus the total throughput for the acknowledgment-based access scheme and for the random-access scheme analyzed in [1].

$r_1 = r_2$, we have found that the optimal p that minimizes the total average delay is always 1. From the steady-state condition, it is easy to see that when $r_1 = r_2 = r$, the total arrival rate γ ($\gamma = 2r$) should be less than 2/3 for steady state. In Fig. 5, the minimal total average delay (i.e., $p = 1$) is plotted versus γ when $r_1 = r_2$ for the access scheme of this paper (the acknowledgment-based access scheme), as well as for the random access scheme analyzed in [1]. As expected and as is shown in Fig. 5, the network performs much better with the access scheme analyzed in this paper. This latter result is actually correct for all values of arrival rates.

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